Revised Syllabus For B.Sc, Part III [Mathematics] (Sem.V &VI)

To be implemented from June 2012.

1. TITLE: Subject Mathematics

2. YEAR OF IMPLEMENTATION: Revised Syllabus will be implemented from June 2012 onwards.

3. DURATION: B.Sc. Part- III The duration of course shall be one year and two semesters.

4. PATTERN: Pattern of examination will be semester.

5. MEDIUM OF INSTRUCTION: English

6. STRUCTURE OF COURSE:

THIRD YEAR B. Sc. (MATHEMATICS) (Semester V & VI)

Semester V

Sr.No	Paper	Name of Paper	Marks	
		Compulsory Theory Papers		
1.	IX	Real Analysis	40 (Theory)	10 (Internal)
2.	X	Modern Algebra	40 (Theory)	10 (Internal)
3.	XI	Partial Differential Equations	40 (Theory)	10 (Internal)
		Optional Theory Papers (Select Any one)		
4.	XII(A)	Symbolic Logic & Graph Theory	40 (Theory)	10 (Internal)
5.	XII(B)	Special Theory of Relativity – I	40 (Theory)	10 (Internal)
6.	XII(C)	Differential Geometry – I	40 (Theory)	10 (Internal)
7.	XII(D)	Mathematical Modeling - I	40 (Theory)	10 (Internal)
8.	XII(E)	Applications of Mathematics in Finance	40 (Theory)	10 (Internal)
9.	XII(F)	Mechanics – I	40 (Theory)	10 (Internal)

Semester VI

Sr.No.	Paper	Name of Paper	Marks	
		Compulsory Theory Papers		
1.	XIII	Metric Spaces	40 (Theory)	10 (Internal)
2.	XIV	Linear Algebra	40 (Theory)	10 (Internal)
3.	XV	Complex Analysis	40 (Theory)	10 (Internal)
		Optional Theory Papers (Select Any one)		
4.	XVI(A)	Algorithms & Boolean Algebra	40 (Theory)	10 (Internal)
5.	XVI (B)	Special Theory of Relativity – II	40 (Theory)	10 (Internal)
6.	XVI (C)	Differential Geometry – II	40 (Theory)	10 (Internal)
7.	XVI (D)	Mathematical Modeling - II	40 (Theory)	10 (Internal)
8.	XVI (E)	Applications of Mathematics in Insurance	40 (Theory)	10 (Internal)
9.	XVI (F)	Mechanics – II	40 (Theory)	10 (Internal)

Practical (Annual Pattern)

COMPUTATIONAL MATHEMATICS LABORATRY – IV **(**Operations Research Techniques**)**	50 Marks
COMPUTATIONAL MATHEMATICS LABORATRY –V **(**Laplace Transform**)**	50 Marks
COMPUTATIONAL MATHEMATICS LABORATRY – VI (Numerical Recipes in C++, Matlab & Microsoft Excel)	50 Marks
COMPUTATIONAL MATHEMATICS LABORATRY –VII **(Project Work, Study Tour, Viva - Voce)**	50 Marks

EQIVALENCE IN ACCORDANCE WITH TITLES AND CONTENTS OF PAPERS (FOR REVISED SYLLABUS)

Sr. No.	Title of Old Paper	Title of New paper
	Compulsory Theory Papers	
1	Mathematics Paper – V (Analysis)	Sem. V :- Real Analysis
		Sem. VI :- Metric Spaces
2	Mathematics Paper – VI (Algebra)	Sem.V :- Modern Algebra
		Sem.VI :- Linear Algebra
3	Mathematics Paper – VII (Complex Analysis & Integral Transform)	Sem. V :- Partial Differential Equations
		Sem. VI :- Complex Analysis
	Optional Theory Papers	
4	Mathematics Paper – VIII(A) (Discrete Mathematics)	Sem. V :- Symbolic Logic & Graph Theory
		Sem. VI :- Algorithms & Boolean Algebra
5	Mathematics Paper – VIII(B) (Special Theory Of Relativity)	Sem.V:- Special Theory of Relativity – I
		Sem.VI:-Special Theory of Relativity II
6	Mathematics Paper – VIII(C) (Differential Geometry)	Sem. V :- Differential Geometry – I
		Sem. VI :- Differential Geometry – II
7	Mathematics Paper – VIII(D) (Mathematical Modeling)	Sem. V :- Mathematical Modeling - I
		Sem.VI :- Mathematical Modeling -II
8	Mathematics Paper – VIII(E) (Application of Mathematics in Finance and Insurance)	Sem. V :- Applications of Mathematics in Finance
		Sem. VI :- Applications of Mathematics in Insurance

Sr.No.	Title of Old Paper	Title of New paper
	Computational Mathematics Laboratory (CML)	
1	Computational Mathematics Laboratory -- IV (Operations Research Techniques)	Computational Mathematics Laboratory -- IV (Operations Research Techniques)
2	Computational Mathematics Laboratory -- V (_Complex Variables and Applications of Differential Equations)	Computational Mathematics Laboratory -- V **(**Laplace Transform**)**
3	Computational Mathematics Laboratory -- VI (Numerical Recipes in C++, Matlab & Microsoft Excel)	Computational Mathematics Laboratory -- VI (Numerical Recipes in C++, Matlab & Microsoft Excel)
4	Computational Mathematics Laboratory -- VII (Project, Viva, Seminar, Tour Report)	Computational Mathematics Laboratory -- VII (Project, Viva, Tour Report)

SHIVAJI UNIVERSITY, KOLHAPUR

B.Sc. Part-III Mathematics

Detail syllabus of semester III and IV

SEMESTER – V

Paper – IX (Real Analysis)

UNIT - 1: SETS AND FUNCTIONS **7 lectures**

1.1 Sets and Elements , Operations on sets

1.2 Functions

1.2.1 Definition of Cartesian product, Function, Extension and restriction of functions, onto function.

1.2.2 THEOREM: If $f: A \to B$ and if $X \subseteq B, Y \subseteq B$, then $f^{-1}(X \cup Y) = f^{-1}(X) \cup f^{-1}(Y)$.

1.2.3 THEOREM: If $f: A \to B$ and if $X \subseteq B, Y \subseteq B$, then $f^{-1}(X \cap Y) = f^{-1}(X) \cap f^{-1}(Y)$.

1.2.4 THEOREM: If $f: A \to B$ and if $X \subseteq A, Y \subseteq A$, then $f(X \cup Y) = f(X) \cup f(Y)$.

1.2.5 THEOREM: If $f: A \to B$ and if $X \subseteq A, Y \subseteq A$, then $f(X \cap Y) = f(X) \cap f(Y)$.

1.2.6 Definition of composition of functions.

1.3 Real – Valued functions

1.3.1 Definition : Real Valued function. Sum, difference, product, and quotient of real valued functions, $\max(f, g)$, $\min(f, g)$, $|f|$. Characteristics function.

1.4 Equivalence, Countability

1.4.1 Definition : one - to - one function, inverse function, 1-1 correspondence and equivalent sets, finite and infinite sets, countable and uncountable set.

1.4.2 Theorem : The countable union of countable sets is countable.

1.4.3 Corollary : The set of rational number is countable.

1.4.4 Theorem : If B is an infinite subset of the countable set A, then B is countable.

1.4.5 Corollary: The set of all rational numbers in [0, 1] is countable.

1.5 Real Numbers

1.5.1 Theorem: The set $[0,1] = \{x : 0 \le x \le 1\}$ is uncountable.

1.5.2 Corollary: The of all real numbers is uncountable.

1.6 Least Upper Bounds

1.6.1 Definition: Upper bound, lower bound of a set, least upper bound.

1.6.2 Least upper bound axiom,

1.6.3 Theorem: If A is any non-empty subset of R that is bounded below, then A has greatest lower bound in R.

UNIT – 2: SEQUENCES OF REAL NUMBERS **13 Lectures**

2.1 Definition of sequence and subsequence

2.2 Limit of a sequence

2.2.1 Definition.

2.2.2 Theorem: If $\{s_n\}_{n=1}^{\infty}$ is a sequence of nonnegative numbers and if $\lim_{n\to\infty} s_n = L$, then L ≥ 0.

2.3 Convergent sequences

2.3.1 Definition

2.3.2 Theorem: If the sequence of real numbers $\{s_n\}_{n=1}^{\infty}$ is convergent to L, then $\{s_n\}_{n=1}^{\infty}$ cannot also converge to a limit distinct from L.That is, if $\lim_{n\to\infty} s_n = L$ and $\lim_{n\to\infty} s_n = M$, then L = M.

2.3.3 Theorem: If the sequence of real numbers $\{s_n\}_{n=1}^{\infty}$ is convergent to L, then any subsequence of $\{s_n\}_{n=1}^{\infty}$ is also convergent to L.

2.3.4 Theorem: All subsequences of real numbers converge to same limit.

2.4 Divergent sequences

2.4.1 Definitions.

2.5 Bounded sequences

2.5.1 Definition.

2.5.2 Theorem: If the sequence of real numbers $\{s_n\}_{n=1}^{\infty}$ is convergent, then $\{s_n\}_{n=1}^{\infty}$ is bounded.

2.6 Monotone sequences

2.6.1 Definition.

2.6.2 Theorem: A non decreasing sequence which is bounded above is convergent.

2.6.3 Theorem: The sequence $\{\left(1+\frac{1}{n}\right)^n\}_{n=1}^{\infty}$ is convergent.

2.6.4 Theorem: A non decreasing sequence which is not bounded above diverges to infinity.

2.6.5 Theorem: A non increasing sequence which is bounded below is convergent.

2.6.6 Theorem: A non increasing sequence which is not bounded below diverges to minus infinity.

2.7 Operations on convergent sequences

2.7.1 Theorem: If $\{s_n\}_{n=1}^{\infty}$ and $\{t_n\}_{n=1}^{\infty}$ are sequences of real numbers, if $\lim_{n\to\infty} s_n = L$ and $\lim_{n\to\infty} t_n = M$, then $\lim_{n\to\infty} (s_n + t_n) = L$ + M. In words the limit of sum (of two convergent sequences) is the sum of the limits.

2.7.2 Theorem: $\{s_n\}_{n=1}^{\infty}$ is of real numbers, if c ϵ R, and if $\lim_{n\to\infty} s_n = L$ then $\lim_{n\to\infty} cs_n = cL$.

2.7.3 Theorem: (a) If 0 < x < 1, then $\{x^n\}_{n=1}^{\infty}$ converges to 0.

(b) If 1< x <∞, then $\{x^n\}_{n=1}^{\infty}$ diverges to infinity.

2.7.4 Theorem: If $\{s_n\}_{n=1}^{\infty}$ and $\{t_n\}_{n=1}^{\infty}$ are sequences of real numbers, if $\lim_{n\to\infty} s_n = L$ and $\lim_{n\to\infty} t_n = M$, then $\lim_{n\to\infty} (s_n - t_n) = L$ – M.

2.7.5 Theorem: If $\{s_n\}_{n=1}^{\infty}$ is sequence of real numbers which converges to L $\{s_n^2\}_{n=1}^{\infty}$ converges to L^2.

2.7.6 Theorem: If $\{s_n\}_{n=1}^{\infty}$ and $\{t_n\}_{n=1}^{\infty}$ are sequences of real numbers, if $\lim_{n\to\infty} s_n = L$ and $\lim_{n\to\infty} t_n = M$, then $\lim_{n\to\infty} s_n t_n = LM$.

2.7.7 Theorem: If $\{t_n\}_{n=1}^{\infty}$ is sequence of real numbers, if and $\lim_{n\to\infty} t_n = M$ where M ≠ 0 then $\lim_{n\to\infty} 1/t_n = 1/M$.

2.7.8 Theorem: If $\{s_n\}_{n=1}^{\infty}$ and $\{t_n\}_{n=1}^{\infty}$ are sequences of real numbers, if $\lim_{n\to\infty} s_n = L$ and $\lim_{n\to\infty} t_n = M$ where M ≠ 0 then $\lim_{n\to\infty} s_n/t_n = L/M$.

2.8 Limit superior and limit inferior

2.8.1 Definition: Limit superior and limit inferior and examples.

2.8.2 Theorem: *If* $\{s_n\}_{n=1}^{\infty}$ is a convergent sequence of the real numbers, then $\text{Lim sup}_{n\to\infty} s_n = \lim_{n\to\infty} s_n$.

2.8.3 Theorem: *If* $\{s_n\}_{n=1}^{\infty}$ is a convergent sequence of the real numbers, then $\text{Lim inf}_{n\to\infty} s_n = \lim_{n\to\infty} s_n$.

2.8.4 Theorem: *If* $\{s_n\}_{n=1}^{\infty}$ is a sequence of the real numbers, then $\text{Lim sup}_{n\to\infty} s_n = \text{lim inf}_{n\to\infty} s_n$.

2.8.5 Theorem: *If* $\{s_n\}_{n=1}^{\infty}$ is a sequence of the real numbers, and if $\text{Lim sup}_{n\to\infty} s_n = \text{lim inf}_{n\to\infty} s_n = L \text{ and } L \in R, \text{ then } \{s_n\}_{n=1}^{\infty}$ is convergent and $\lim_{n\to\infty} s_n = L$.

2.8.6 Theorem: $If\ \{s_n\}_{n=1}^{\infty}$ is a sequence of the real numbers, and if

$\limsup_{n\to\infty} s_n = \liminf_{n\to\infty} s_n = \infty$, $then\ \{s_n\}_{n=1}^{\infty}$ diverges to infinity.

2.8.7 Theorem: $If\ \{s_n\}_{n=1}^{\infty}$ and $\{t_n\}_{n=1}^{\infty}$ are bounded sequences of real numbers, and if

$s_n \le t_n (n \in I)$, $then\ \limsup_{n\to\infty} s_n \le \limsup_{n\to\infty} t_n\ and\ \liminf_{n\to\infty} s_n \le \liminf_{n\to\infty} t_n.$

2.8.8 Theorem: $if\ \{s_n\}_{n=1}^{\infty}$ and $\{t_n\}_{n=1}^{\infty}$is are bounded sequences of real numbers, then

$\limsup_{n\to\infty} (s_n + t_n) \le \limsup_{n\to\infty} s_n + \limsup_{n\to\infty} t_n$; $\liminf_{n\to\infty} (s_n + t_n) \ge \liminf_{n\to\infty} s_n + \liminf_{n\to\infty} t_n.$

2.8.9 Theorem (Statement Only): Let $\{s_n\}_{n=1}^{\infty}$ be bounded sequences of real numbers,

a) If $\limsup_{n\to\infty} s_n = M$, then for any $\varepsilon > 0$

(i) $s_n < M + \varepsilon$ for all but a finite number of values of n;

(ii) $s_n > M - \varepsilon$ for infinitely many values of n;

b) if $\liminf_{n\to\infty} s_n = M$, then for any $\varepsilon > 0$,

c) $s_n > M - \varepsilon$ for all but finite number of values of n;

d) $s_n < M + \varepsilon$ for infinitely many values of n;

2.8.10 Theorem: Any bounded sequences of real numbers have a convergent subsequence.

2.9 Cauchy sequences

2.9.1 Definition.

2.9.2 Theorem: If the sequence of real numbers $\{s_n\}_{n=1}^{\infty}$ converges, then $\{s_n\}_{n=1}^{\infty}$ is a Cauchy sequence.

2.9.3 Theorem: If $\{s_n\}_{n=1}^{\infty}$ is Cauchy sequence of real numbers, then $\{s_n\}_{n=1}^{\infty}$ is bounded.

2.9.4 Theorem: If $\{s_n\}_{n=1}^{\infty}$ is Cauchy sequence of real numbers, then $\{s_n\}_{n=1}^{\infty}$ is convergent.

2.9.5 Theorem: If for each $n \in I$ let I_n = [a_n, b_n] be a (nonempty) closed bounded interval of real numbers such that

(a) $I_1 \supseteq I_2 \supseteq \ldots \supseteq I_n \supseteq I_{n+1} \supseteq \ldots$

(b) $\lim_{n\to\infty} (b_n - a_n) = \lim_{n\to\infty} (length\ of\ I_n) = 0$. Then $\bigcap_{n=1}^{\infty} I_n$ contains precisely one point.

2.10 Summability of sequences

2.10.1 Definition: (C, 1) Summability and examples.

2.10.2 Theorem: $If\ \lim_{n\to\infty} s_n = L,\ then\ \lim_{n\to\infty} s_n = L\ (C, 1).$

UNIT– 3: SERIES OF REAL NUMBERS. **12 Lectures**

3.1 Convergence and divergence.

3.1.1 Definition: Series.

3.1.2 Definition: Convergence of series.

3.1.3 Theorem: If $\sum_{n=1}^{\infty} a_n$ converges to A and $\sum_{n=1}^{\infty} b_n$ converges to B, then $\sum_{n=1}^{\infty}(a_n + b_n)$ converges to A + B. Also if $c \in R$, then $\sum_{n=1}^{\infty} ca_n$ converges to cA.

3.1.4 Theorem: If $\sum_{n=1}^{\infty} a_n$ is convergent series, then $\lim_{n\to\infty} a_n = 0$.

3.2 Series with non-negative terms.

3.2.1 Theorem: If $\sum_{n=1}^{\infty} a_n$ is series of non-negative numbers with $s_n = a_1 + \cdots + a_n (n \in I)$, then

a) $\sum_{n=1}^{\infty} a_n$ converges if the sequence $\{s_n\}_{n=1}^{\infty}$ is bounded.

b) $\sum_{n=1}^{\infty} a_n$ diverges if $\{s_n\}_{n=1}^{\infty}$ is not bounded.

3.2.2 Theorem: a) If 0 < x <1 then $\sum_{n=0}^{\infty} x^n$ converges to 1/ (1-x).

b) If $x \ge 1$, then $\sum_{n=0}^{\infty} x^n$ diverges.

3.2.3 Theorem: The series $\sum_{n=1}^{\infty}(1/n)$ is divergent.

3.3 Alternating series.

3.3.1Theorem: If $\{a_n\}_{n=1}^{\infty}$ is sequence of positive numbers such that

a) $a_1 \ge a_2 \ge \cdots \ge a_n \ge a_{n+1} \ge \cdots$, and

b) $\lim_{n\to\infty} a_n = 0$,

then the alternating series $\sum_{n=1}^{\infty}(-1)^{n+1} a_n$ is convergent.

3.4 Conditional convergence and absolute convergence.

3.4.1 Definition.

3.4.2 Theorem: If $\sum_{n=1}^{\infty} a_n$ converges absolutely, then $\sum_{n=1}^{\infty} a_n$ converges.

3.4.3 Theorem: a) If $\sum_{n=1}^{\infty} a_n$ converges absolutely then both $\sum_{n=1}^{\infty} p_n$ and $\sum_{n=1}^{\infty} q_n$ converges.

b) If $\sum_{n=1}^{\infty} a_n$ converges conditionally, then both $\sum_{n=1}^{\infty} p_n$ and $\sum_{n=1}^{\infty} q_n$ diverge.

3.5 Tests for absolute convergence.

3.5.1 Definition.

3.5.2 Theorem: If $\sum_{n=1}^{\infty} a_n$ is dominated by $\sum_{n=1}^{\infty} b_n$ where $\sum_{n=1}^{\infty} b_n$ converges absolutely, then $\sum_{n=1}^{\infty} a_n$ converges absolutely.

3.5.3 Theorem: If $\sum_{n=1}^{\infty} a_n$ is dominated by $\sum_{n=1}^{\infty} b_n$ and $\sum_{n=1}^{\infty} |a_n| = \infty$, then $\sum_{n=1}^{\infty} |b_n| = \infty$.

3.5.4 Theorem: a) $\sum_{n=1}^{\infty} b_n$ converges absolutely and if $\lim_{n\to\infty} |a_n|/|b_n|$ exists, then $\sum_{n=1}^{\infty} a_n$ converges absolutely.

b)If $\sum_{n=1}^{\infty} |a_n| = \infty$ and if $\lim_{n\to\infty} |a_n|/|b_n|$ exists, then $\sum_{n=1}^{\infty} |b_n| = \infty$.

3.5.5 Theorem: Let $\sum_{n=1}^{\infty} a_n$ be a series of nonzero real numbers and let

$a = \liminf_{n\to\infty} \left|\frac{a_{n+1}}{a_n}\right|, A = \limsup_{n\to\infty} \left|\frac{a_{n+1}}{a_n}\right|$, then

a) If $A < 1$, then $\sum_{n=1}^{\infty} |a| < \infty$;

b) If $a > 1$, then $\sum_{n=1}^{\infty} a_n$ diverges;

c) If $a \leq 1 \leq A$, then the test fails.

3.5.6 Theorem: If $\limsup_{n\to\infty} \sqrt[n]{|a_n|}$ =A then the series of real numbers $\sum_{n=1}^{\infty} a_n$

a) Converges absolutely if $A < 1$,

b) Diverges if $A > 1$. If $A = 1$ the test fails.

3.5.7 Theorem: Let $\{a_n\}_{n=1}^{\infty}$ be a sequence of real numbers. Then

a) If $\limsup_{n\to\infty} \sqrt[n]{|a_n|}$ =0, the series $\sum_{n=0}^{\infty} a_n x^n$ converges absolutely for all real x.

b) If $\limsup_{n\to\infty} \sqrt[n]{|a_n|} = L > 0$, then series $\sum_{n=0}^{\infty} a_n x^n$ converges absolutely for $|x| < \frac{1}{L}$ and diverges if $|x| > \frac{1}{L}$;

c) If $\limsup_{n\to\infty} \sqrt[n]{|a_n|} = \infty$ then $\sum_{n=0}^{\infty} a_n x^n$ converges only for $x = 0$ and diverges for all other x.

3.6 Series whose terms form a non-increasing sequence

3.6.1 Theorem: *If* $\{a_n\}_{n=1}^{\infty}$ is the non increasing sequence of positive numbers and *if* $\sum_{n=0}^{\infty} 2^n a_{2^n}$ converges, then $\sum_{n=1}^{\infty} a_n$ converges. Examples.

3.6.2 Theorem: *If* $\{a_n\}_{n=1}^{\infty}$ is the non increasing sequence of positive numbers and *if* $\sum_{n=0}^{\infty} 2^n a_{2^n}$ diverges, then $\sum_{n=1}^{\infty} a_n$ diverges. Examples.

3.6.3 Theorem: The series $\sum_{n=1}^{\infty} \frac{1}{n^2}$ converges.

3.6.4Theorem: *If* $\{a_n\}_{n=1}^{\infty}$ is a non increasing sequences of positive numbers and if $\sum_{n=1}^{\infty} a_n$ Converges , then $\lim_{n\to\infty} n a_n = 0$. Examples.

3.7 The Class l^2

3.7.1 Definition of the class l^2.

3.7.2 Theorem: The Schwarz inequality.

3.7.3 Theorem: Minkowski inequality.

3.7.4 Norm of an element in l^2.

3.7.5 Theorem: The norm for sequences in l^2 has the following properties:

$\|x\|_2 \geq 0 \quad (x \in \mathbb{R}^n)$, $\|x\|_2 = 0$ if and only of $x = \{0\}_{i=1}^{n}$,

$\|\alpha x\|_2 = |\alpha| . \|x\|_2 \quad (\alpha \in R, x \in \mathbb{R}^n)$, $\|x + y\|_2 \leq \|x\|_2 + \|y\|_2 \quad (x, y \in \mathbb{R}^n)$.

UNIT – 4 RIEMANN INTEGRATION. 13 Lectures

4.1 Riemann integrability & integrals of bounded functions over bounded intervals:

4.1.1 Definitions and simple examples: Subdivision and norm of subdivision, lower & upper sums, lower & upper integrals, oscillatory sum, Riemann integral.

4.2 Darboux's Theorem:

4.2.1 Lemma: Let f(x) be a function defined on [a , b] for which there is a $k \in R$ such that |f(x)| ≤ k. Let D_1 be a subdivision of [a , b] and D_2 be the subdivision of [a, b] consisting of all points of D_1 and at the most p more, with $|D_1| \leq \delta. Then S(D_1) - 2pk\delta \leq S(D_2) \leq S(D_1)$.

4.2.2 Theorem: To every $\epsilon > 0$, there corresponds $\delta > 0$ such that $S(D) < \int_a^b f(x)\,dx + \epsilon$, for every D with $|D| \leq \delta$.

4.2.3 Theorem: To every $\epsilon > 0$, there corresponds $\delta > 0$ such a that $s(D) > \int_a^b f(x)\,dx - \epsilon$, for every D with $|D| \leq \delta$.

4.2.4 Theorem: For every bounded function f on [a, b], prove that upper integral ≥ the lower integral.

4.3 Equivalent definition of integrability and integrals.

4.3.1 Theorem: If f is bounded and integrable over[a, b] , that to $\epsilon > 0$ there corresponds $\delta > 0$, such a that for every subdivision $D = \{a = x_0, x_1, x_2, \ldots\ldots, x_n = b\}$ with $|D| \leq \delta$ and for every choice of $\xi_r \in [x_{r-1}, x_r]$, $\left|\sum_{r=1}^{n} f(\xi_r)(x_r - x_{r-1}) - \int_a^b f(x)\,dx\right| < \epsilon$

4.3.2 Theorem: If f is integrable over [a, b] and if there exists a number I such a that to every $\epsilon > 0$ there correspond $\delta > 0$ such a that for every sub division $D = \{a = x_0, x_1, x_2, \ldots\ldots, x_n = b\}$ with $|D| \leq \delta$ and for every choice of with $\xi_r \in [x_{r-1}, x_r]$, $\left|\sum_{r=1}^{n} f(\xi_r)(x_r - x_{r-1}) - I\right| < \epsilon$ then I is value of $\int_a^b f(x)\,dx$.

4.4 Conditions for integrability.

4.4.1Theorem: the necessary and sufficient condition for the integrability of a bounded function f over [a, b] is that to every $\epsilon > 0$, there corresponds

$\delta > 0$ such that for every subdivision D of [a, b] with $|D| \le \delta$, the oscillatory sum w(D)<ε.

4.4.2 Theorem: the necessary and sufficient condition for the integrability of a bounded function f over [a, b] is that to every $\varepsilon > 0$, there corresponds a subdivision D of [a, b] such a that the corresponding oscillatory sum w(D) < ε.

4.5 Particular classes of bounded integrable functions:

4.5.1Theorem: Every continuous function on [a, b] is Riemann

4.5.2 Theorem: Every monotonic function on [a, b] is Riemann integrable.

4.5.3 Theorem: Every bounded function on [a, b] which has only a finite number of points of discontinuities is Riemann integrable.

4.5.4 Theorem: If function f is bounded on [a, b] and the set of all points of discontinuities has a finite number of limit points then f is Riemann integrable over [a, b].

4.6 Properties of integrable functions:

4.6.1 Theorem: If a bounded function f is integrable on [a, b] then f is also integrable on [a, c] & [c, b], for a < c < b and conversely. In this case

$$\int_a^b f(x)\,dx = \int_a^b f(x)\,dx + \int_a^b f(x)\,dx$$

4.6.2 Lemma: The oscillation of a bounded function f on [a, b] is the list upper bound of the set$\{|f(\alpha) - f(\beta)| \alpha, \beta \in [a, b]\}$.

4.6.3 Theorem: If f & g are both bounded and integrable functions on [a, b] then f ± g are also bounded &integrable over [a, b] and

$$\int_a^b [f(x) \pm g(x)]dx = \int_a^b f(x)\,dx \pm \int_a^b g(x)\,dx$$

4.6.4 Theorem: If f & g are both bounded and integrable functions on [a, b] then the product f.g is also bounded &integrable over [a, b].

4.6.5 Theorem: If f & g are both bounded and integrable functions on [a, b] and if there exist $t > 0$ with $|g(x)| \ge t, (a \le x \le b)$ then $\frac{f}{g}$ is also bounded and integrable over [a, b].

4.6.6 Theorem: If f is both bounded and integrable function on [a, b] then | f | is also bounded &integrable over [a, b].

4.7 Inequalities for an integral:

4.7.1 Theorem: If f is bounded and integrable function on [a, b] and if M and m are the least upper and greatest lower bounds of f over [a, b] then

$$m(b-a) \leq \int_a^b f(x)\,dx \leq m(b-a), if\, a \leq b\, and$$

$$m(a-b) \geq \int_a^b f(x)\,dx \geq m(a-b), if\, b \leq a.$$

4.7.2 Theorem: If f is bounded and integrable over [a, b] with $|f(x)| < k$

$$then \left| \int_a^b f(x)\,dx \right| \leq k\,(b-a)$$

4.7.3 Theorem: $If \int_a^b |f(x)|\,dx$ exist then $\left|\int_a^b f(x)\,dx\right| \leq \int_a^b |f(x)|\,dx$

4.8 Function defined by a definite integral:

4.8.1 Definition: the integral function of an integrable function f on [a, b].

4.8.2 Theorem: The integral function of an integrable function is continuous.

4.8.3 Theorem: The integral function φ, of a continuous function f, is continuous and

$$\varphi' = f$$

4.9 Theorems of Integral Calculus (statements only):

4.9.1 Fundamental Theorem of Integral calculus.

4.9.2 First Mean Value Theorem of Integral calculus.

4.9.3 Second Mean Value Theorem of Integral calculus.

4.9.4 Integration by Change of variable.

4.9.5 Integration by Parts.

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2. T. M. Apostol, Mathematical Analysis, Narosa Publishing House
3. Satish Shirali, H. L. Vasudeva, Mathematical Analysis, Narosa Publishing House.
4. D. Somasundaram, B. Choudhary, First Course in Mathematical Analysis, Narosa Publishing House
5. W. Rudin, Principles of Mathematical Analysis, McGraw Hill Book Company.
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8. Malik, Arrora, Mathematical Analysis, Wiley Eastern Ltd.

Paper – X (Modern Algebra)

Unit – 1 :GROUPS **15 lectures**

1.1 Preliminaries: Divisibility in integers, the greatest common divisor (g. c. d.), function from a set A to a set B, a 1-1 function, an onto function, prime and composite numbers, congruence relation on the set of integers, Permutations, Cyclic Permutations, Transpositions, Disjoint Permutations, Even and odd permutations.

1.1.1 Theorem: (without proof): Euclid's algorithm: If m is a positive integer and n is any integer then there exists integers q and r such that $n = mq + r$, where $0 \leq r < m$.

1.1.2 Theorem: (without proof): For any two non-zero integers a & b, there exists a greatest common divisor d of a and b such that $d = ax + by$ for some integers x and y.

1.1.3Theorem: (without proof): Two integers a & b are relatively prime if and only if there exist two integers x & y such that $ax + by = 1$.

1.2 Group and commutative group: Definitions & examples.

1.2.1 Theorems: In a group G

a) Identity element is unique.

b) Inverses of each $a \in G$ is unique.

c) $(a^{-1})^{-1} = a$, for all $a \in G$.

d) $(ab)^{-1} = b^{-1}a^{-1}$ for all $a, b \in G$.

e) $ab = ac \Rightarrow b = c$, $ba = ca \Rightarrow b = c$ for all $a, b, c \in G$.

1.2.2 Theorem: For elements a, b in a group G, the equations $ax = b$ and $ya = b$ have unique solutions for x and y in G.

1.3 Definition of Subgroupand examples

1.3.1 Theorem: A non empty subset H of a group G is subgroup of G iff

i) $a, b \in H \Rightarrow ab \in H$ ii) $a \in H \Rightarrow a^{-1} \in H$

1.3.2 Theorem: A non-empty subset H of a group G is subgroup of G iff

$a, b \in H \Rightarrow ab^{-1} \in H$

1.4 Definition of Centre of group G and Normalizer of a in G.

1.4.1 Theorem: Centre of group G is a subgroup of G

1.4.2 Normalizer is a subgroup of G.

1.5 Definition of left and right cosets and $Ha = \{x \in G \mid x \equiv a \bmod H\} = cl(a)$ for any $a \in G$.

1.6 Definition of order of group.

1.6.1 Lagrange's Theorem: If G is finite group and H is subgroup of G then o(H) divides o(G).

1. 6. 2 Theorem: Ha = H if and only if a Є H.

1.6.3 Theorem: Ha = Hb if and only if ab^{-1} Є H.

1.6.4 Theorem: Ha is a subgroup of G if and only if a ЄH.

1.6.5 Theorem: HK is subgroup of G iff HK = KH.

1.7 Definition of Cyclic group and Order of element of a group.

1.7.1 Theorem: Order of a cyclic group is equal to the order of its generator.

1.7.2 Theorem: A subgroup of a cyclic group is cyclic.

1.7.3 Theorem: A cyclic group is abelian.

1.7.4 Theorem: If G is finite group, then order of any element of G divides order of group G.

1.7.5 Theorem: An infinite cyclic group has precisely two generators.

1.7.6 Definition of Eule'sϕ function.

1.7.7 Theorem: Number of generators of a finite cyclic group of order n is $\phi(n)$.

1.7.8 Euler's Theorem: Let a, n ($n \geq 1$) be any integers such that g.c.d. (a, n) = 1. Then $a^{\phi(n)} \equiv 1 \pmod{n}$.

1.7.9 Fermat's Theorem: For any integer a and prime p, $a^p \equiv a \pmod{p}$ and examples.

Unit – 2: NORMAL SUBGROUPS, HOMOMORPHISMS, PERMUTATION GROUP

10 Lectures

2.1 Normal Subgroups: Definition and Examples.

2.1.1 Theorem: A subgroup H of a group G is normal in G iff $g^{-1}Hg = H$ for all $g \in G$.

2.1.2 Theorem: A subgroup H of a group G is normal in G iff $g^{-1}hg \in H$ for all $h \in H$, $g \in G$.

2.1.3 Theorem: A subgroup H of a group G is normal in G iff product of two right cosets of H in G is again a right coset of H in G.

2.1.4 Definition of Quotient Group.

2.1.5 Theorem: If G is finite group and N is a normal subgroup of G then

$$o\left(\frac{G}{N}\right)=\frac{o(G)}{o(N)}$$

2.1.6 Theorem (without proof) : Every quotient group of a cyclic group is cyclic

2.2 Definitions of Homomorphism, Isomorphism, Epimorphism, Monomorphism, Endomorphism, and Automorphism.

2.2.1 Theorem: If $f:G\to G'$ is homomorphism then f(e) = e'

2.2.2 Theorem: If $f:G\to G'$ is homomorphism then $f(x^{-1}) = [f(x)]^{-1}$

2.2.3 Theorem: If $f:G\to G'$ is homomorphism then $f(x^n) = [f(x)]^n$, n an integer.

2.2.4 Definition of Kernel of homomorphism.

2.2.5 Theorem: If $f:G\to G'$ is homomorphism then Ker f is a normal subgroup of G.

2.2.6 Theorem: A homomorphism $f:G\to G'$ is one-one iff Ker f = {e}.

2.2.7 Fundamental Theorem of group homomorphism: If $f:G\to G'$ is an onto homomorphism with K = Ker f, then $\frac{G}{K}\cong G'$

2.2.8 Second Theorem of isomorphism: Let H and K be two subgroups of group G, where H is normal in G, then $\frac{HK}{H}\cong\frac{K}{H\cap K}$

2.2.9 Third Theorem of isomorphism: If H and K be two normal subgroups of group G, such that H $\subseteq$ K then $\frac{G}{K}\cong\frac{G/H}{K/H}$

2.2.10 Cayley's Theorem: Every group G is isomorphic to a permutation group.

Unit –3: RINGS **8 Lectures**

3.1 Definition of Ring, Commutative ring, Zero divisor, Integral Domain Division Ring, Field, Boolean ring.

3.1.1 Theorem: A field is an integral domain.

3.1.2 Theorem: A non-zero finite integral domain is field.

3.2 Definition of Subring

3.2.1Theorem: A non-empty subset S of ring R is a subring of R if and only if a, b $\in$ S $\Rightarrow$ab, a – b $\in$ S

3.3 Characteristic of a ring.

3.3.1 Theorem: Let R be ring with unity. If 1 is additive order n then ch R = n. if 1 is of additive order infinity then ch R is 0.

3.3.2 Theorem: If D is an integral domain, then characteristic of D is either zero or prime number.

3.3.3 Definition of Nilpotent, Idempotent, product of rings.

3.4 Definition of Ideal

3.4.1 Definition of Sum of two ideals.

3.4.2 Theorem: If A and B of two ideals of R then A + B is an ideal of R containing both A and B.

3.5 Definition of Simple Ring.

3.5.1 Theorem: A division is a simple ring.

Unit –4: HOMOMORPHISM AND IMBEDDING OF RING **12 Lectures**

4.1 Definition of Quotient Rings, Homomorphism, Kernel of homomorphism.

4.1.1 Theorem: If $\theta : R \to R'$ be a homomorphism, then $\theta(0) = 0'$

4.1.2 Theorem: If $\theta : R \to R'$ be a homomorphism, then $\theta(-a) = -\theta(a)$

4.1.3 Theorem: If $f : R \to R'$ is homomorphism then Ker f is an ideal of R.

4.1.4 Theorem: If $f : R \to R'$ is homomorphism then Ker f = $\{0\}$ if and only if f is one –one.

4.1.5 Fundamental Theorem of ring homomorphism: If $f : R \to R'$ is an onto homomorphism with K = Ker f, then $\frac{R}{K} \cong R'$

4.1.6 First Theorem of isomorphism: Let B $\subseteq$ A be two ideal of ring R. then $\frac{R}{A} \cong \frac{R/B}{A/B}$

4.1.7 Second Theorem of isomorphism: Let A, B be two ideals of ring R then $\frac{A+B}{A} \cong \frac{B}{A \cap B}$

4.2 Definition of Imbedding ring.

4.2.1 Theorem: Any ring can be imbedded into a ring with unity.

4.3 Definition of Maximal Ideal and Prime Ideal.

4.3.1 Theorem: Let R be a commutative ring with unity. An ideal M of R is maximal ideal of R iff $\frac{R}{M}$ is a field.

4.3.2 Theorem: Let R be a commutative ring. An ideal P of R is prime iff $\frac{R}{P}$ is an integral domain.

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3. **Fundamentals of Abstract Algebra,** Malik D. S. Morderson J.N. and Sen M. K. McGraw Hill, 1997.

4. **A Text Book of Modern Abstract Algebra,** Shanti Narayan

5. **Modern Algebra,** Surjeet Sing and Quazi ZameeruddinVikas Publishing House, 1991.

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7. **Basic Algebra Vol. I & II ,** N. Jacobson ,W. H. Freeman 1980.

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Paper – XI (Partial Differential Equations)

Unit: 1. Linear Partial Differential Equations of Order One : 10 Lectures

1.1 Explanation of the terms :

1. Partial differential Equation.
2. Order of the Partial differential equation.
3. Degree of the Partial differential equation
4. Linear Partial Differential equation.

1.2 Derivation of a partial differential equation by the elimination of arbitrary constants.

1.3 Derivation of a partial differential equation by the elimination of arbitrary function & from the equation  (u,v) = 0 where u and v are the functions of x, y and z.

1.4 Lagrange's Linear Partial Differential Equation.

1.5 Lagrange's method of solving the linear partial differential equation P p + Qq = R of order one.

1.6 Working Rule for Solution of Langranges linear. Partial differential equation Pp + Qq = R where P, Q, R are functions of x, y and z.

1.7 Geometrical Interpretation of Langranges linear partial differential equation.

Unit 2 : Non-Linear Partial Differential Equations of order one . 10 Lectures

2.1 Explanation of the terms.

1. Non linear partial differential equation.
2. Solution or Integral of a partial differential equation.
3. Complete Integral.
4. Particular Integral.
5. General Integral.
6. Singular Integral.

2.2 Special Methods of Solutions applicable to some standard forms.

2.2.1 Standard Form I : Partial differential equations of the form f (p,q) – 0

2.2.2 Standard Form II : Clairauts form Z = px + qy + f (p,q) = 0

2.2.3 Standard Form III : Partial differential equations of the form f (z,p,q) =0

2.2.4 Standard Form IV: Partial differential equations of the form

$f_1(x, p) = f_2(y, q)$

2.3 General Method of solving equations of order one but of any degree: Charpit's Method.

2.4 Working Rule for Charpit's Method.

Unit 3: Linear Homogeneous Partial Differential Equations with Constant coefficients.

12 Lectures.

3.1 Explanation of the terms:

1. Linear partial differential equation of order n.
2. Solution of Linear Partial differential equation.
3. Linear Homogeneous Partial differential equation with constant coefficients

3.2 Solution of linear homogeneous partial differential equation with constant coefficients of the form $F(D,D')\ z = f\ (x,y)$ where $F\ (D,D')$ is a homogeneous function of D and D'.

3.3 Methods for finding the complementary functions: (C.F)

3.4 Methods for finding the particular Integrals (P.I)

3.5 Finding the particular methods for Integral when $f(x, y)$ is of the form $\varnothing^n\ (ax + by)$ when $f\ (a, b) = 0$ and $f\ (a, b) \neq 0$

3.6 General method for finding particular Integral.

Unit 4: Non-Homogeneous Linear Partial Differential Equations with Constant Coefficients **13 Lectures**

4.1 Explanation of the terms:

4.1.1. Non – homogeneous linear partial differential equation with constant coefficients.

4.1.2. Solution of non homogeneous partial differential equation of the form $F\ (D, D')\ z = f\ (x,y)$ where $F\ (D,D')$ is non homogeneous function of D and D'.

4.2 Solution of the equation $(D\text{-}m\ D'\text{-}K)\ z = 0$

4.3 Methods for finding the complementary function (C.F) of a non homogeneous equation $F(D,D')\ z = 0$ where $F(D,D')$ can be factorized in to linear factors of D and D'.

4.4 Methods for finding particular Integral (P.I) of non homogeneous linear equations with constant coefficients of the form $F\ (D,D')\ z = f\ (x,y)$.

Case I : When $f\ (x,y) = e^{ax + by}$ and $F\ (a,b) \neq 0$

Case II : When $f\ (x,y) = \sin\ (ax + b)$ or $\cos\ (ax + b)$

Case III : When $f\ (x,y) = x^m y^m$

Case IV : When $f\ (x,y) = Ve^{ax + by}$, where V is a function of x and y.

4.5 Method for finding complementary function of a non homogeneous equation $F(D,D')\,z = 0$ where $F(D,D')$ can not be factorized in to linear factors of D and D'.

4.6 Equations reducible to linear form with Constant coefficients.

REFERENCE BOOKS

1. Differential Equations: Gupta – Malic – Mittal., Pragati Prakashan Meerut.
2. Differential Equations: Sharma – Gupta., Krishna Prakashan Meerut.
3. Ordinary and Partial Differential Equations : M.D. Raisinghnia S–Chand and Coy–New Delhi.

(Optional Papers)

Paper XII (A) (Symbolic Logic and Graph Theory.)

Unit – 1 :Arguments and The Method of Deduction: 10 Lectures

1.1: The nature of Argument: Concept of Inference, Argument, conclusions and premises, deductive arguments, inductive arguments, valid and invalid arguments.

1.2: Formal proof of validity: Limitations of truth tables, Formal proof of validity, Rules of Inference: Modus Ponens, Modus Tollens, Hypothetical Syllogism, Disjunctive Syllogism, Constructive Dilemma, Destructive Dilemma, Simplification, Conjunction, Addition, Method of proving the validity.

1.3: The Rule of Replacement or The Principle of Extensionality: De Morgan's Laws, Commutation, Association, Distribution, Double Negation, Transposition, Material Implication, Material Equivalence, Exportation, Tautology. Directions for the constructions of a formal proof of validity.

1.4: Invalidity and Incompleteness: Proving Invalidity, The Rule of Conditional proof, The Rule of Indirect proof (The Method of proof by *reductio adabsurdum*) Proofs of Tautologies: Use of the rule of Conditional Proof, Use of the Rule of Indirect Proof. The Strengthened Rule of Conditional Proof.

Unit - 2 : Propositional Functions and Quantification Theory 12 Lectures

2.1: Definitions of Singular and General Propositions with illustrations, Symbolizing Singular Propositions, Propositional Function and itssubstitution instance, instantiation process.

2.2: Quantification (or Generalization) universal quantifier, an existential quantifier Negation of a propositional function and a singular proposition, Negation of quantifiers, Relationship between universal and existential quantification.

2.3: Types of subject – predicate propositions: Universal Affirmative, Universal Negative, Particular Affirmative, Particular Negative; and relation between them.

2.4: Preliminary Rules of Quantificational Deductions: Universal Instantiation (U.I.), Universal Generalization (U.G.), Existential Generalization (E.G.) Existential Instantiation (E.I.). Procedure of Quantificational Deduction and examples. Use of conditional proof for proving the validity of a valid argument involving quantifiers.

Unit – 3 :GRAPHS **12 lectures**

3.1 Basic Terminology: Definitions & simple examples of Directed and undirected graphs, isomorphism of two graphs, a sub – graph and the complement of a graph, the spanning sub-graph, directed & undirected complete graphs, multi – graphs & weighted graphs, a finite-state model of simple situations, the linear graphs, paths & circuits, connected graphs.

3.1.1 **Theorem:** In a graph with n vertices if there is a path from vertex v_1 to v_2 then there is a path of no more than (n – 1) edges from v_1 to v_2.

3.2 Shortest path in a weighted graph : Definitions.

3.2.1 Algorithm: (E.W.Dijkstra's algorithm): To find shortest path in a weighted graph & examples.

3.3 Eulerian paths & circuits: Eulerian path & circuit, the degree of a vertex in an undirected graph, the statement of the hand-shaking lemma, the incoming & the outgoing degree of a vertex in a directed graph, the statement of the Hand-shaking dilemma.

3.3.1 Theorem: If an undirected graph possesses an Eulerian path then it is connected and has either no vertex of odd degree or exactly two vertices of odd degree.

3.3.2 Theorem: If an undirected connected graph has exactly two vertices of odd degree then it possesses an Eulerian path.

3.3.3 Theorem: (statement only): An undirected graph possesses an Eulerian circuit if and only if it is connected and its vertices are all of even degree.

3.4 Hamiltonian paths & circuits: Definitions.

3.4.1 Theorem: If a linear undirected graph G has n vertices and if the sum of the degrees for each pair of vertices in G is greater than or equal to (n-1) then G is connected.

3.4.2 Theorem: (Statement only): If a linear undirected graph G has n vertices and if the sum of the degrees for each pair of vertices in G is greater than or equal to (n–1) then there exists a Hamiltonian path in G.

3.4.3 Theorem: (Statement only): There is always a Hamiltonian path in a directed complete graph.

3.5 The traveling salesperson problem (TSP): Definitions.

3.5.1 Algorithm: The nearest neighbour method to solve the given TSP which gives reasonably good answer to the TSP & examples.

3.6 Planar graphs: a planar graph, finite & infinite regions.

3.6.1 Theorem: (Euler's formula): For any connected planar graph, prove: $v - e + r = 2$, where v, e, and r are the number of vertices, edges, and regions of the graph respectively.

3.6.2 Theorem: Prove: $e \leq 3v - 6$, in any connected linear planar graph that has no loops and has two or more edges.

3.6.3 Theorem: Prove that: $e \leq 2v - 4$, in a planar graph in which every region would be bounded by four or more edges. 4) Examples: The complete graphs K_5& $K_{3,3}$ are not planar.

Unit – 4 :TREES **11 lectures**

4.1 Trees: Definitions & simple examples.

4.1.1 **Theorem:** (Properties of trees):

i)There is a unique path between every two vertices in a tree.

ii) The number of vertices is one more than the number of edges in a tree.

iii) A tree with two or more vertices has at least two leaves.

4.1.2 Theorem: (Characterizations of trees): A graph in which there is a unique path between every pair of vertices is a tree.

4.1.3 A connected graph in which the number of vertices is one more than the number of edges is a tree.

4.1.4 A graph without circuit in which the number of vertices is one more than the number of edges is a tree.

4.2 Rooted trees: a directed tree, a rooted tree, an ordered rooted tree, isomorphism of ordered rooted trees, binary trees, regular trees, the path length of a vertex & the height of a tree, prefix codes, a binary tree for the weights w_1, w_2,…, w_n& the weight of it, an optimal tree, pre-fix & post-fix notations for an algebraic expression.

4.3 **Applications:**

4.3.1 Representation of the algebraic expressions as ordered binary trees, use them to write expressions in pre – fix & post – fix notations.

4.3.2 The relationship between i, the number of branch nodes and t, the number of leaves of a regular binary trees.

4.3.3 The bounds of the number of leaves of an m-array tree of height h.

4.3.4 The D. A. Huffman procedure for construction of an optimal tree for a given set of weights.

4.4 Spanning trees & cut – sets: a spanning tree of a connected graph, a cut-set of a connected graph, the fundamental circuit & cut–set of a connected graph relative to the spanning tree of the graph.

4.5 Properties of circuits & cut–sets

4.4.1 Theorem: A circuit and the complement of any spanning tree must have at least one edge in common.

4.4.2 Theorem: A cut–set and any spanning tree must have at least one edge in common.

4.4.3 Theorem: Every circuit has an even number of edges in common with every cut–set.

4.5 Minimum spanning trees: Definitions.

4.5.1 Algorithm: To determine a minimum spanning tree, MST, of a connected weighted graph & examples.

4.6 Transport networks: Definitions.

4.6.1 Theorem: The value of any flow in a given transport network is less than or equal to the capacity of any cut in the network.

4.6.2 Algorithm: (Labeling procedure): To find a maximum flow in a transport network & examples.

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6. Discrete Mathematical Structures (Third Edition), Bernard Kolman, Robert C. Busby, Sharon Ross, Prentice Hall of India private Limited, (2001), New Delhi-110 001.

Paper – XII (B) (Special Theory of Relativity – I)

Unit – 1 : CLASSICAL THEORY OF RELATIVITY (10 lectures)

1.1 Review of Newtonian Mechanics.

1.1.1 Inertial system.

1.1.2 Event.

1.2 Galilean Transformations.

1.2.1 Newtonian Relativity.

1.2.2 Conservation laws in Newtonian Mechanics.

1.2.3 Ether.

1.2.4 Maxwell′s electromagnetic theory.

1.3 The Michelson – Morley experiment.

1.3.1 Fitzgerald and Lorentz Contraction hypothesis.

Unit – 2 : LORENTZ TRANSFORMATIONS (13 lectures)

2.1 Einstein′s Special Relativity Theory.

2.1.1 Einstein′s principle of relativity.

2.1.2 Principle of constancy of light speed.

2.2 Lorentz Transformations.

2.3 Consequences of Lorentz Transformation

2.3.1 Lorentz – Fitzgearld length contraction

2.3.2 Time dilation

2.3.3 Clock paradox or twin paradox.

2.3.4 Simultaneity

2.4 Geometrical Interpretation of LT.

2.5 Group property of Lorentz Transformations and examples

Unit – 3 : RELATIVISTIC KINEMATICS (7 lectures)

3.1 Introduction.

3.2 Transformation of particle velocity.

3.3 Relativistic addition law for velocities.

3.4 Transformation of the Lorentz contraction factor $(1 - v^2/c^2)^{1/2}$.

3.5 The Transformations for the acceleration of a particle.

Unit – 4 : RELATIVISTIC MECHANICS (15 lectures)

4.1 Introduction (Mass and Momentum). $m = \dfrac{m_0}{\sqrt{1-\dfrac{u^2}{c^2}}}$

4.2 The mass of a moving particle

4.2.1 Relativistic expression for Force.

4.2.2 Transverse and Longitudinal mass of the particle.

4.3 Mass energy equivalence $E = mc^2$

4.4 Transformation equations for mass.

4.5 Transformation equations for momentum and energy.

4.5.1 Deduction to prove that $p^2 - E^2/c^2$ is Lorentz invariant.

4.6 Minkowski Space (Four Dimensional Continuum).

4.6.1 Time-like, Space- like, Light – like (null) intervals.

4.6.2 Events occurring at the same point and the same time.

4.6.3 Theorem : There exists an inertial system S' in which the two events occur at one and the same point if the interval between two events is timelike.

4.6.4 Corollary : Two events which are separated by a timelike interval cannot occur simultaneously in any inertial system.

4.6.5 Theorem : There exists an inertial system S' in which the two events occur at one and the same time if the interval between two events is spacelike.

4.6.6 World points and World lines.

4.6.7 Lorentz transformations in index form.

4.7 Past, Present, Future – Null Cone.

4.7.1 Proper time.

REFERENCE BOOKS

1. Special Relativity, T. M. Karade, K. S. Adhav and Maya S. Bendre, SonuNilu , 5,BanduSoni Layout , Gayatri Road, Parsodi, Nagpur, 440022.
2. Theory of Relativity (Special and General), J.K.Goyal , K.P.Gupta, Krishna Prakashan Media (P) Ltd., Meerut., 2006.
3. Relativity and Tensor Calculus, Karade T. M. Einstein Foundation International, 1980.
4. Mechanics, Landau L. D. and Lifshitz E. M.,Butterworth, 1998.
5. The Theory of Relativity, Moller C., Oxford University Press, 1982.

Paper XII(C) (Differential Geometry I – (Curves and Surfaces))

Unit – 1: SPACE CURVES **12 lectures**

1.1 Parametric equations for space curves.

1.2 Vector representation of a curve.

1.3 Function of class r.

1.4 Path.

1.4.1 Equivalent paths.

1.4.2 Change of parameters.

1.5 Arc Length.

1.5.1 Arc Length of curve between two points.

1.5.2 Cartesian form.

1.5.3 Examples.

1.6 Tangent lines.

1.6.1 Equation of tangent line to a curve at a point.

1.6.2 Examples.

1.7 The osculating plane.

1.7.1 Definition 1, Definition 2.

1.7.2 Find the equation of osculating plane using Definition 1 & Definition 2.

1.8 The tangent plane at any point of the surface $f(x, y, z) = 0$.

1.8.1 Normal plane.

1.8.2 Normal plane is perpendicular to the osculating plane.

1.8.3 The osculating plane at a point of space curve given by the Intersection of surfaces $f(r) = 0$; $\psi(r) = 0$.

1.8.4 Examples.

1.9 The principal Normal and Binomial (Definitions).

1.9.1 The direction of principal normal and binomial.

1.9.2 The unit vectors t, n, b.

1.9.3 Equations principal normal and binomial.

Unit – 2: CURVETURE **13 lectures**

2.1 Definition (Curvature and Torsion).

2.2 Screw Curvature (Definition).

2.3 SerretFrenet Formulae.

2.4 SerretFrenet Formula (incartesion form).

2.5 Examples.

2.6 Find curvatures and torsion of curves.

2.6.1 When the equation of curve is r = f (u).

2.6.2 When the equation of curve is r = f(S).

2.7 Examples (Curvature and Torsion).

2.8 Involutes and Evolute (Definitions).

2.8.1 Involute of space curve.

2.8.2 Find curvature κ_1 and torsion τ_1 of the involute.

2.8.3 Evolute of space curve.

2.8.4 Find curvature and torsion of the evolute.

Unit – 3: FUNDAMENTAL FORMS **10 lectures**

3.1 First fundamental form or metric.

3.2 Geometrical interpretation of metric.

3.3 Properties I and II.

3.4 Second fundamental form.

3.5 Geometrical Interpretation.

3.6 Examples.

Unit – 4 :DERIVATIVE OF N **10 lectures**

4.1 Derivative of N (The surface normal); Weingarten Equations.

4.2 Examples.

4.3 Angle between two parametric curves.

4.3.1 Necessary and Sufficient condition for the parametric curves for the surface to be orthogonal.

4.3.2 Through every point of the surface there passes one and only one parametric curves of each system.

4.4 Direction coefficients (Def^n).

4.4.1 To find the angle between two directions on a surface at a point p.

4.4.2 Direction ratios (Def^n).

4.4.3 Relation between ⊚, ,m and (λ, μ).

REFERENCE BOOKS

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Prentice – hall 1976.

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University Press 1965.

Paper XII(D) (Mathematical Modeling – I)

Unit – 1 : MATHEMATICAL MODELLING : NEED, TECHNIQUES, CLASSIFICATION AND SIMPLE ILLUSTRATION 10 lectures

1.1 Simple situations requiring Mathematical Modeling.

1.2 The technique of Mathematical Modeling.

1.3 Classification of Mathematical Models.

1.4 Some characteristics of Mathematical Models.

1.5 Mathematical Modeling through Geometry.

1.6 Mathematical Modeling through Algebra.

1.7 Mathematical Modeling through Trigonometry.

1.8 Limitations of Mathematical Modeling.

Unit –2: MATHEMATICAL MODELLING THROUGH ORDINARY DIFFERENTIAL EQUATIONS OF FIRST ORDER 12 lectures

2.1 Mathematical Modeling through differential equations.

2.2 Linear Growth and Decay Models.

2.3 Non-linear Growth and Decay Models.

2.4 Compartment Models.

2.5 Mathematical Modeling in Dynamics through Ordinary differential Equations of First Order.

Unit – 3 : MATHEMATICAL MODELLING THROUGH SYSTEMS OF ORDINARY DIFFERENTIAL EQUATIONS OF FIRST ORDER 12 lectures

3.1 Mathematical Modeling in Population Dynamics.

3.2 Mathematical Modeling of Epidemics through System of Ordinary differential equations of First order.

3.3 Compartment Models through Systems of Ordinary Differential Equations.

3.4 Mathematical Modeling in Economics through System of Ordinary Differential Equations of First order.

3.5 Mathematical Models in Medicine, Arms Race Battles and Internationals Trades in terms of System of Ordinary Differential Equations.

Unit – 4 : MATHEMATICAL MODELLING THROUGH ORDINARY DIFFERENTIAL EQUATIONS OF SECOND ORDER **11 lectures**

4.1 Mathematical Modeling of Planetary Motions.

4.2 Mathematical Modeling of Circular Motion and Motion of Satellites.

4.3 Mathematical Modeling through Linear Differentials Equations of Second Order.

4.4 Miscellaneous Mathematical Models through Ordinary Differential Equations of the Second Order.

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1. Mathematical Modeling, J. N. Kapur, New Age International (P) Ltd., Publishers Reprint 2003.
2. Differential Equations and Their Application ,ZafarAhsan ,Prentice Hall of India , Delhi
3. Mathematical Modeling, J.G. Andrews and R. R. Mclone (1976). Butterwerths London.
4. Mathematical Modeling Techniques, R. Aris (1978) , Pitman.
5. Differential Equation Models, Martin Braun, C. S. Coleman, D.A.Drew , Vol. 1.
6. Political and Related Models, Steven J. Drams, Kl. F Lucas, P. D. Straffin (Eds),Vol. 1.
7. Discrete and System Models, W. F. Lucas, F. S. Roberts, R. M. Thrall, Vol. 3.
8. Life Science Models, H. M. Roberts And M. Thompson, Vol. 4.
9. “Thinking with Models” (Mathematical Models in Physical, Biological and Social Sciences), T. Saaty and J.Alexander Pergamon Press, New York.

Paper XII(E) - (Applications of Mathematics in Finance)

Unit – 1 :FINANCIAL MANAGEMENT **10 lectures**

1.1 An overview.

1.2 Nature and Scope of Financial Management.

1.3 Goals of Financial Management and main decisions of financial management.

1.4 Difference between risk, speculation and gambling.

Unit – 2 :TIME VALUE OF MONEY **20 lectures**

2.1 Interest rate and discount rate.

2.2 Present value and future value,

2.3 discrete case as well as continuous compounding case.

2.4 Annuities and its kinds.

2.5 Meaning of return.

2.5.1 Return as Internal rate of Return (IRR).

2.5.2 Numerical Methods like NewtonRaphson Method to calculate IRR.

2.5.3 Measurement of returns under uncertainty situations.

2.6 Meaning of risk.

2.6.1 Difference between risk and uncertanity.

2.6.2 Types of risks. Measurements of risk.

2.7 Calculation of security and Portfolio Risk and Return- Markowitz Model.

2.7.1 Sharpe's Single Index Model.

2.7.2 Systematic Risk and Unsystematic Risk.

Unit – 3 :TAYLOR SERIES AND BOND VALUATION **5 lectures**

3.1 Calculation of Duration and Convexity of bonds.

Unit – 4 :FINANCIAL DERIVATIVES. **10 lectures**

4.1 Futures. Forward.

4.2 Swaps and Options.

4.3 Call and Put Option.

4.3.1 Call and Put Parity Theorem.

4.4 Pricing of contingent claims through Arbitrage and Arbitrage
Theorem.

REFERENCE BOOKS

1. Corporate Finance -Theory and Practice, Aswath Damodaran, John Wiley & Sons.Inc.
2. Options, Futures, and Other Derivatives, John C. Hull, Prentice-Hall of India Private Limited.
3. An Introduction to Mathematical Finanace, Sheldon M. Ross, Cambridge University Press.
4. Introduction to Risk Management and Insurance, Mark S. Dorfman,Prentice Hall, Englwood Cliffs, New Jersey

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PAPER NO XII (F) – (MECHANICS- I)

1. Friction: Laws of friction, angle of friction, cone of friction, least force required to pull the body up and down on the rough inclined plane **12 lectures**
2. Common catenary: different forms of equations, geometrical properties of the common catenary, catenary of uniform strength. **12 lectures**
3. Impact: Direct impulse, law of linear momentum for the impulse, Newton's experimental law, coefficient of restitution, impulse of the blow, oblique impact, loss of kinetic energy due to impact. **11 lectures**
4. Virtual work: principle of virtual work, moment of force, couple, conditions of equilibrium of a rigid body. **10 lectures**

REFERENCE BOOKS

1. Statics by A.S. Ramsey, CBS Publishing and distributors
2. Mechanics by B.Singh, S.K. Pundir, P.K.Sharma, Pragatiprakashan
3. Vectors and mechanics by K.M.Agashe, Ram Chand and co Delhi Dynamics by M. Ray

SEMESTER – VI

Paper – XIII (METRIC SPACES)

UNIT –1: LIMITS AND METRIC SPACES **8 lectures**

1.1 Revision: Limits of a function on the real line.

1.2 Metric space: Definition :of Metric space and examples inclusive of each of $R^1, R_d, R^n, l^\infty, l^2,$

1.3 Limits in metric spaces

1.3.1 Definition of $\lim_{x\to a} f(x) = L$.

1.3.2 If $\lim_{x\to a} f(x) = L$ and $\lim_{x\to a} g(x) = N$ then

i) $\lim_{x\to a}[f(x) + g(x)] = L + N;$

ii) $\lim_{x\to a}[f(x) - g(x)] = L - N;$

iii) $\lim_{x\to a}[f(x)\, g(x)] = LN$ and

iv) $\lim_{x\to a}[f(x) / g(x)] = \frac{L}{N}$ $(N \neq 0)$

1.4 Definition: Sequences and their convergence in metric space, Cauchy sequence in metric space.

1.4.1 Theorem : A sequence of points in any metric space cannot converge to two distinct limits

1.4.2 Theorem : Every convergent sequence in metric space is Cauchy.

1.4.3 Example to illustrate that every Cauchy sequence need not be convergent.

1.4.4 Theorem : Every Cauchy sequence of real numbers is bounded.

1.4.5 Theorem : If a Cauchy sequence has a convergent subsequence then the sequence itself is convergent.

1.4.6 Theorem : Every Cauchy sequence in R_d is convergent.

UNIT - 2: CONTINUOUS FUNCTIONS ON METRIC SPACES **15 lectures**

2.1 Functions continuous at a point on the real line.

2.1.1 Definition: Continuity of a function

2.1.2 Theorem (Statement only) : If real valued functions f and g are continuous at $a \in R^1,$ then so are $f + g,$ $f - g,$ $fg,$ $\frac{f}{g},$ $f \circ g,$ $cf,$ $|f|$ where $c \in R$ at $a.$

2.2 Reformulation:

2.2.1 Theorem : The real valued function f is continuous at $a \in R^1$ if an only if given $\epsilon > 0$ there exists $\delta > 0$ such that $|f(x) - f(a)| < \epsilon \quad (|x - a| < \delta)$.

2.2.2 Definition: The open ball of radius r about a.

2.2.3 Theorem : The real valued function f is continuous at $a \in R^1$ if and only if the inverse image under f of any open ball $B[f(a); \epsilon]$ about $f(a)$ contains an open ball $B[a;\delta]$ about a.

2.2.4 Theorem : A function f is continuous at a , if and only if $\lim_{n\to\infty} x_n = a \Rightarrow \lim_{n\to\infty} f(x_n) = f(a)$.

2.3 Functions continuous on a metric space

2.3.1 Definition: The open ball of radius r about a in a metric space.

2.3.2 Definition: Continuity of function defined on a metric space

2.3.3 Theorem : The function f is continuous at $a \in M_1$ if and only if any one of the following conditions hold

i) Given $\epsilon > 0$, there exist $\delta > 0$ such a that $\rho_2[(f(x), f(a)] < \epsilon \quad (\rho_1(x, a) < \delta)$

ii) The inverse image under f of any open ball $B[f(a); \epsilon]$ about $f(a)$ contains an open ball $B[a;\delta]$ about a.

iii)Whenever $\{x_n\}_{n=1}^{\infty}$ is a sequence of points in M_1 converging to a, then the sequence $\{f(x_n)\}_{n=1}^{\infty}$ of point in M_2 converging to $f(a)$

2.3.4 Theorem : If f is continuous at $a \in M_1$ and g is continuous at $f(x) \in M_2$, then $g \circ f$ is continuous at a.

2.3.5 Theorem : Let M be a metric space, and let f and g be real valued functions which are continuous at $a \in M$, then so are $f + g, \ f - g, \ fg, \frac{f}{g}, |f|$ at a.

2.3.6 Definition of continuity of a function $f : M_1 \to M_2$.

2.3.7 Theorem : If f and g be continuous functions from a metric space M_1 into a matrix space M_2 , then so are $f + g, \ f - g, \ fg, \frac{f}{g}, |f|$ on M_1.

2.4 Open sets.

2.4.1 Definition: Open set

2.4.2 Any open ball in a metric space is an open set.

2.4.3 Theorem : In any metric space $\langle M, \rho \rangle$,both M and $\emptyset$ are open sets.

2.4.4 Theorem : Arbitrary union of open sets is open.

2.4.5 Theorem : Every subset of R_d is open.

2.4.6 Theorem : Finite intersection open sets is open.

2.4.7 Theorem : Every open subset G of R^1 can be written as $G = \cup I_n$ Where $I_1, I_2, \ldots$ *are a finite number or a countable number of open* intervals which are mutually disjoint.

2.4.8 Theorem : A function is continuous if and only if inverse image of every open set is open.

2.5 Closed sets

2.5.1 Definition: Limit point, closure of a set.

2.5.2 Theorem : If E is any subset of the metric space M, then $E \subset \bar{E}$.

2.5.3 Definition: Closed set.

2.5.4 Theorem : Let E be a subset of the metric space M . Then the point $x \in M$ is a limit point of E if and only if every open ball $B[x; r]$ about x contains at least one point of E .

2.5.6 Theorem : In an y metric space both M and ϕ are closed sets.

2.5.7 Theorem : Arbitrary intersection of closed sets is closed.

2.5.8 Theorem : Finite union of closed sets is closed.

2.5.9 Theorem :Let G be an open subset of the metric space M . Then $G' = M - G$ is closed. Conversely, if f is closed subset of M, then $F' = M - F$ is open.

2.5.10 Theorem : Let $\langle M_1, \rho_1 \rangle$ and $\langle M_2, \rho_2 \rangle$ be metric spaces., and let $f : M_1 \to M_2$. Then f is continuous on M_1 if and only if $f^{-1}(F)$ is a closed subset of M_2.

2.5.11 Theorem :Let f be a 1-1 function from a metric space M_1 onto a matrix space M_2. Then if f has any one of the following properties, it has then all.

i)Both f and f^{-1} are continuous

ii)The set $G \subset M_1$ is open if and only if its image $f(G)$ subset M_2 is open.

iii)The set F $\subset M_1$ is closed if and only if its image $f(F)$ is closed.

2.5.12 Definition : Homeomorphism, dense subset of a metric space.

2.5.13 Show that R^1 and R_d are not homeomorphic.

UNIT-3: CONNECTEDNESS, COMPLETENESS, & COMPACTNESS 15 lectures

3.1 More about open sets

3.1.1 Theorem : Let , $< M, \rho >$ be a metric space and let A be a subset of M . Then the subset G_A is open subset of $< A, \rho >$ if and only if there exist an open subset G_M of $< M, \rho >$ such a that $G_A = A \cap G_M$.

3.2 Connected sets:

3.2.1 Theorem : Let , $< M, \rho >$ be a metric space and let A be a subset of M Then if A be a subset of M. Then if A has either one of the following properties it has the other.

a)It is impossible to find nonempty subsets A_1, A_2 of M such that $A = A_1 \cup A_2$, $\overline{A_1} \cap A_2 = \emptyset$, $A_1 \cap \overline{A_2} = \emptyset$.

b) When $< A, \rho >$ is itself regarded as metric space, then there is no set expect A and $\emptyset$ which is both open and closed in $< A, \rho >$.

3.2.2 Definition: Connected set

3.2.3 Theorem : The subset A of R^1 *is connected if and only if whenever* $a \in A, b \in A$ with $a < b$, then $c \in A$ *for* any c such a that $a < c < b$.

3.2.4 Theorem : A continuous function carries connected sets to connected sets.

3.2.5 Theorem : If f is a continuous real valued function on the closed bounded interval [a, b], then f takes on every value between $f(a)$ and $f(b)$.

3.2.6 Theorem : A metric space is connected if and only if every continuous characteristic function on it is constant.

3.2.7 Theorem : If $A_1 \cap A_2 \neq \emptyset$, then $A_1 \cup A_2$ is also connected.

3.2.8 Theorem : The interval [0,1] is not connected sunset of R_d.

3.3 Bounded and totally bounded sets.

3.3.1 Definition: Bounded subset of metric space, totally bounded sets.

3.3.2 Theorem : Every totally bounded set is bounded.

3.3.3 Theorem : A subset A of R_d is totally bounded if and only if A contains only a finite number of points.

3.3.4 Definition: e - dense set.

3.3.5 Theorem : The subset A of the metric space $< M, \rho >$ is totally bounded if and only if, for every $\varepsilon > 0$, A contains a finite subset $\{x_1, x_2, \ldots, x_n\}$ which is ε - dense in A.

3.3.6 Theorem :Let $< M, \rho >$ be a metric space. The subset A of M is totally bounded if and only if every sequence of points of A contains a Cauchy sequence.

3.4 Complete metric space.

3.4.1 Definition: Complete metric space.

3.4.2 Theorem : If $< M, \rho >$ be a complete metric space, and A is a closed subset of M, then $< A, \rho >$ is also complete.

3.4.3 Generalized nested interval theorem.

3.4.4 Definition: Contraction operator.

3.4.5 Theorem : Let $< M, \rho >$ be a complete metric space. If T is a contraction on M, then there is one and only one point x in M such That $Tx = x$.

3.4.6 R_d *is complete and* R^2 *is complete.*

3.5 Compact metric spaces

3.5.1 Definition: Compact metric space.

3.5.2 The metric space $< M, \rho >$ is compact if and only if every sequence of points in M has a subsequence converging to a point in M.

3.5.3 Theorem : A closed subset of a compact metric space is compact.

3.5.4 Theorem : Every compact subset of a metric space is closed.

3.5.5 Definition: Covering and open covering

3.5.6 Theorem : If M is a compact metric space, then M has the Heine- Borel property.

3.5.7 Theorem : If a metric space M has Heine-Borel property, the M is compact.

3.5.8 Definition: Finite intersection property.

3.5.9 Theorem : The metric space M is compact if and only if, whenever $\mathcal{F}$ is a family of closed subsets of M with finite intersection property, then $\bigcap_{F \in \mathcal{F}} F \neq \emptyset$.

3.5.10 Theorem : Finite subset of any metric space is compact.

UNIT- 4 : SOME PROPERTIES OF CONTINUOUS FUNCTIONS ON METRIC SPACE **7 lectures**

4.1 Continuous functions on compact metric space

4.1.1 Theorem : Let f be a co ntinuous function of the compact metric space M_1. Then the range $f(M_1)$ of f is also compact.

4.1.2 Theorem : Let f be a continuous function of the compact metric space M_1 into a metric space M_2. Then the range $f(M_1)$ of f is a bounded subset of M_2.

4.1.3 Definition: Bounded function

4.1.4 Theorem : If the real valued function f is continuous on a closed bounded interval in R^1, then f must be bounded.

4.1.5 Theorem : If the real valued function f is continuous on the compact metric space M, then f attains a maximum value at some point of M.

4.1.6 Theorem : If the real valued function f is continuous on a closed bounded interval [a, b] then f *attains a maximum and minimum* value at some point of [a, b].

4.1.7 Theorem : If f is a continuous real valued function on the compact connected metric space M, then f takes on every value between its minimum value and its maximum value.

4.2 Uniform continuity

4.2.1 Definition: Uniform continuity.

4.2.2 Let $< M_1, \rho_1 >$ be a compact metric space. If f is a continuous function from M_1 *into a metric space* $< M_2, \rho_2 >$ then f is uniformly continuous of M_1.

4.3.3 If the real valued function f is continuous on the closed bounded interval [a, b] then f *is uniformly continuous on[a, b].*

4.3.4 Let $< M_1, \rho_1 >$ a metric space and let A be a dense subset of M_1. If f is a uniformly continuous function from $< A, \rho_1 >$ into a complete metric space $< M_2, \rho_2 >$, ten f can be extended to a uniformly continuous function F from M_1 into M_2.

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4. D. Somasundaram, B. Choudhary, First Course in Mathematical Analysis, Narosa Publishing House

5. W. Rudin, Principles of Mathematical Analysis, McGraw Hill Book Company.

6. Shantinarayan, Mittal, A Course of Mathematical Analysis, S.Chand and Company.

7. J.N. Sharma, Mathematical Analysis-I, Krishna Prakashan Mandir, Meerut.

8. Malik, Arrora, Mathematical Analysis, Wiley Eastern Ltd.

Paper XIV (Linear Algebra)

Unit –1: VECTOR SPACES **18 Lectures**

1.1 Definition of vector space and simple examples.

1.2 Theorem: In any vector space V(F) the following results hold

i) $0 \cdot x = 0$

ii) $\alpha \cdot 0 = 0$

iii) $(-\alpha)x = -(\alpha x) = \alpha(-x)$

iv) $(\alpha - \beta)x = \alpha x - \beta x$

1.3 Definition of subspace

1.4 Theorem: A necessary and sufficient condition for a non empty subset W of a vector space V(F) to be a subspace is that W is closed under addition and scalar multiplication.

1.5 Theorem: A non empty subset W of a vector space V(F) is a subspace of V if and only if $\alpha x + \beta y \in W \; for \, \alpha, \beta \in F, x, y \in W$.

1.6 Definition of sum of subspaces, direct sum, and quotient space, homomorphism of vector space and examples.

1.7 Theorem: Under a homomorphism $T : V \to U$

i) T(0) = 0

ii) T(-x) = -T(x)

1.8 Definition of Kernel and Range of homomorphism.

1.9 Theorem : Let $T : V \to U$ be a homomorphism, then Ker T is a subspace of V.

1.10 Theorem: Let $T : V \to U$ be a homomorphism, then Ker T = {0} if and only if T is one – one.

1.11 Theorem: Let $T : V \to U$ be a L.T. (linear transformation) then range of T is a subspace of U

1.12 Theorem: Let W be a subspace of V, then there exists an onto L.T. $\theta : V \to \frac{V}{W}$ such that Ker θ=W

1.13 Definition of Linear Span.

1.14 Theorem: L(S) is the smallest subspace of V containing S.

1.15 Theorem: If W is subspace of V then L(W) = W and conversely.

1.16 Definition of Finite dimensional vector space (F. D.V. S), Linear Dependence and independence, basis of vector space and examples.

1.17 Theorem: If S = $\{v_1, v_2, v_3, \ldots, v_n\}$ is a basis of V then every element of V can be expressed uniquely as a linear combination of $v_1, v_2, v_3, \ldots, v_n$.

1.18 Theorem: Suppose S is a finite subset of a vector space V such that V = L(S) then there exists a subset of S which is a basis of V.

1.19 Definition of F.D.V.S.

1.20 Theorem: If V is a F.D.V.S. and $\{v_1, v_2, v_3, \ldots, v_r\}$ is a L.I. subset of V, then it can be extended to form a basis of V.

1.21 Theorem: If dim V = n and S = $\{v_1, v_2, v_3, \ldots, v_n\}$ spans V then S is a basis of V.

1.22 Theorem: If dim V = n and S = $\{v_1, v_2, v_3, \ldots, v_n\}$ is a L.I. subset of V then S is a basis of V.

Unit –2: INNER PRODUCT SPACES **10 Lectures**

2.1 Definition of Inner product space, norm of a vector and examples.

2.2 Theorem: Cauchy-Schwarz inequality. Let V be an inner product space. Then $|(u, v)| \leq \|u\| \|v\|$, for all u, v Є V.

2.3 Theorem: Triangle inequality. Let V be an inner product space. Then $\| u + v \| \leq \| u \| + \| v \|$, for all u, v Є V.

2.4 Theorem: Parallelogram law. Let V be an inner product space. Then $\| u + v \|^2 + \| u - v \|^2 = 2 (\| u \|^2 + \| v \|^2)$, for all u, v Є V.

2.5 Definition of Orthogonal vectors and orthonormal sets.

2.6 Theorem: Let S be a orthogonal set of non-zero vectors in an inner product space V. Then S is a linearly independent set.

2.7 Gram-Schmidt orthogonalisation process

2.7.1 Theorem : Let V be a non trivial inner product space of dimension n. Then V has an orthonormal basis.

2.7.2 Examples.

Unit –3: LINEAR TRANSFORMATION **10 Lectures**

3.1 Definition of L.T., Rank and Nullity and Examples.

3.2 Theorem : A L.T. $T : V \rightarrow V$ is one – one iff T is onto.

3.3 Theorem : Let V and W be two vector spaces over F. Let {v_1, v_2, v_3, . . . ,v_n}be a basis of V and w_1, w_2, w_3, . . ., w_n be any vectors in W (not essentially distinct). Then there exists a unique L.T. $T: V \to W \; s.t. T(V_i) = w_i \;\; i = 1,2,...n$

3.4 Theorem: (Sylvester's Law) : Suppose V and W are finite dimensional vector spaces over a field F. Let T: V→W be a linear transformation. Then rank T + nullity T = dim V.

3.5 Theorem: If $T: V \to V$ be a L.T. Show that the following statements are equivalent.

i) Range T $\cap$ Ker T = {0}

ii) If T(T(v)) = 0 then T(v) = 0, v $\in$V.

3.6 Definition of Sum and Product of L.T., Linear operator, Linear functional and examples.

3.7 Theorem: Let T, T_1, T_2 be linear operators on V and let $I: V \to V$ be the identity mapping I(v) = v for all v then

i) I T = T I = T

ii) $T(T_1 + T_2) = TT_1 + TT_2$, $(T_1 + T_2)T = T_1T + T_2T$

iii) $\alpha(T_1T_2) = (\alpha T_1)T_2 = T_1(\alpha T_2) \quad \alpha \in F$

iv) $T_1(T_1T_2) = (T_1T_2)T_1$

3.8 Definition of Invertible L.T. and examples.

3.9 Theorem: A L.T. $T: V \to W$ is a nonsingular iff T carries each L.T. subset of V onto a L.I. subset of W.

3.10 Theorem: Let $T: V \to W$ be a L.T. where V and W are F.D.V.S. with same dimension. Then the following are equivalent.

i) T is invertible.

ii) T is nonsingular.

iii) T is onto.

3.11 Theorem: Let $T: V \to W$ and $S: W \to U$ be two L.T. Then

i) If S and T are one – one onto then ST is one-one onto and $(ST)^{-1} = T^{-1}S^{-1}$.

ii) If ST is one – one then T is one-one.

iii) If ST is onto then S is onto.

3.12 Definition of Matrix of L.T. and examples.

3.12.1.**Theorem:** $Hom(U,V) \cong M_{m\times n}(F)$.

3.13 Definition of Dual space.

3.13.1 Theorem: Let V be n dimensional vector space over a field F. Then the dimension the dual space of V over F is n.

Unit –4: <u>EIGEN VALUES AND EIGEN VECTORS</u> **7 Lectures**

4.1 Definition of Eigen values, Eigen vectors, Eigen space of order n.

4.1.1 examples.

4.2 Let A be a square matrix of order n. If λ is an eigen value of A then the set all eigen vectors of A corresponding to λ together with zero vector, forms a subspace of n dimensional unitary space.

4.3 Theorem: Let A be a square matrix of order n having k distinct eigen values $\lambda_1, \lambda_2, \ldots, \lambda_k$. Let v_i be an eigen vector corresponding to the eigen value λ_i, i = 1, 2, …,k. Then the set $\{v_1, v_2, \ldots v_k\}$ is linearly independent.

4.4 Theorem: Let A be a square matrix of order n having n distinct eigen values $\lambda_1, \lambda_2, \ldots, \lambda_k$. Let v_i be an eigen vector corresponding to the eigen value λ_i, i = 1, 2, …,n. Then the set $\{v_1, v_2, \ldots v_n\}$ is basis for the domain space of A. The matrix of the linear transformation A with respect to the basis $\{v_1, v_2, v_3, \ldots v_n\}$ is

$$\begin{bmatrix} \lambda_1 & 0 & 0 & \ldots & 0 \\ 0 & \lambda_2 & 0 & \ldots & 0 \\ 0 & 0 & \lambda_3 & \ldots & 0 \\ \ldots & \ldots & \ldots & \ldots & \\ 0 & 0 & 0 & \ldots & \lambda_n \end{bmatrix}$$

4.5 Examples on application of 4.4.

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6. **Abstract Algebra, T**. M. Karade, J. N. Salunkhe, K. S. Adhav, M. S. Bendre,SonuNilu ,Einstein Foundation International, Nagpur 440022.

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8. **A Text Book of Modern Algebra, R**. Balakrishnan, N. Ramabhadran, Vikas Publishing House Pvt. Ltd., 1996.

9. **Linear Algebra ,**Dr. GanadharParia, New Central Book Agency, 1st edition, 1992.

10. **Linear Algebra ,**Surjit Singh, Vikas Publishing House Pvt. Ltd., 1978.

Paper – XV (COMPLEX ANALYSIS)

Unit – 1: ANALYTIC FUNCTIONS **13 lectures**

1.1 Limits and continuity of a function of a complex variable.

1.2 Differentiable function of a complex variable.

1.2.1 Differentiability and continuity

1.2.2 Elementary rules of differentiation

1.3 Analytic function and Analytic function in domain

1.4 Necessary and sufficient condition for f(z) = u + iv to be analytic.

1.5 Examples on 1.4

1.6 Polar form of Cauchy – Riemann Partial differential equations.

1.7 Harmonic function, conjugate harmonic function.

1.8 Construction of analytic function f(z) = u + ivby

(a) Milne-Thomson Method

(b) Determining conjugate function.

1.9 Examples on 1.7 and 1.8.

Unit – 2: COMPLEX INTEGRATION **17 lectures**

2.1 Jordan curve, Orientation of Jordan curve.

2.2 Simply connected and multiply connected domains.

2.3 Rectifiable curves and their properties.

2.4 Integral along an oriented curve, line integral.

2.5 Properties

(a) $\int_C [f(z) + g(z)]\, dz = \int_C f(z)\, dz + \int_C g(z)\, dz$

(b) If C has two parts C_1, C_2 then $\int_C f(z)dz = \int_{C_1} f(z)\, dz + \int_{C_2} f(z)\, dz$

(c) $\int_C f(z)dz = -\int_{-C} f(z)\, dz$

2.6 Theorem (without proof) “If C is defined by $z = z(t)$ is continuously differentiable and $f(z)$ is continuous over C, then the curve C is rectifiable and $\int_C f(z)dz = \int_a^b [u(t) + iv(t)]\,dt$.”

2.7 Examples on line integrals 2.4

2.8 Cauchy′s Integral Theorem and proof by using Green′s theorem.

2.9 Extension of Cauchy′s Integral Theorem to multiply connected domains.

2.10 Cauchy′s Integral Formula for f(a).

2.11 Cauchy′s Integral Formula for multiply connected domain.

2.12 Higher order derivatives of an analytic function

2.13 Examples on 2.10 to 2.12

2.14 Development of an analytic functions as a power series

(a) Taylor's Theorem for complex functions

(b) Statement of Laurent series

(c) Theorem (without proof) " Every function $f(z)$ analytic in a doubly connected domain D defined by $\rho < |z-a| < R$ which is a circular ring with centre a is expressible as a Laurent series so that there exist a relation of the form $f(z) = \sum_{-\infty}^{\infty} a_n (z-a)^n$, where $a_n = \frac{1}{2\pi i}\int_c \frac{f(\xi)}{(\xi-a)^{n+1}} d\xi$."

(d) Examples on Taylor's and Laurent series

Unit – 3: SINGULAR POINT AND CALCULUS OF RESIDUES **8 lectures**

3.1 Zeros of an analytic function

3.2 Singularities of an analytic function, Types of Singularities.

3.3 Examples on 3.1, 3.2.

3.4 Residue at an isolated singularity and Residue at infinity.

3.5 Computation of residues.

3.6 Cauchy′s residue theorem.

3.7 Examples on Cauchy′s residue theorem.

Unit – 4: EVALUATION OF INTEGRALS **7 lectures**

4.1 Evaluation of the integral $\int_0^{2\pi} f(\cos\theta, \sin\theta) d\theta$

4.2 Statement of lemma, "If $f(z)$ is such that $zf(z) \to 0$ uniformly as $z \to \infty$, then $\lim_{R\to\infty} \int_{C_1} f(z)dz = 0$, where C_1 is a semi-circle $|z| = R, I(z) > 0$."

4.3 Evaluation of the integral $\int_{-\infty}^{\infty} f(x)dx$, where $f(x)$ is a real rational function of the real variable x.

4.4 Statement of Jorden's lemma

4.5 Evaluation of the integral $\int_{-\infty}^{\infty} f(x)\, sin\, mx dx$ and $\int_{-\infty}^{\infty} f(x)\, cos\, mx dx$

REFERENCE BOOKS

1. Theory of Functions of a Complex Variable. Shanti Narayan, P.K.Mittal, S.Chand & Company LTD. 2005 (Revised 8th edition), Ram Nagar, New Delhi – 110055.

2. Complex Variables and Applications,R. V. Churichill& J. W. Brown, 5th Edition, McGraw – Hill, New – York, 19.

3. Complex Variables Introduction and Applications, Mark J. Abtowitz& A. S. Fokas, Cambridge University Pre SouthSouth Asian Edition, 1998

4. Mechanics & Complex Variables, Karade T. M.,Ajantha Arts International, 1998.

5. Lectures on Real Analysis, Karade T. M. and Bendre M. S., SonuNilu, EinsteinFoundation International, Nagpur, 2002.

6. Complex Variables, M. L. Khanna, Jai PrakashNath& Co., Meerut, (U. P.)

7. Complex Variables, J. N. Sharma, Krishna PrakashaqnMandir, Subhsh Bazar, Meerut – 2, (U. P.).

(Optional Papers)

Mathematics Paper-XVI (A) (Algorithms and Boolean Algebra)

Unit – 1 :Finite state machines & algorithms **11 lectures**

1.1 Definitions & simple examples: Information processing machine with & without memory, finite state machine, tabular & graphical description of finite state machine.

1.2 Examples of finite state machines: Modeling of Physical systems.

1.3 Finite state machines as language recognizer: Use of FSM as Language recognizer, Definition of finite state language.

1.4 Examples: Finite state languages & languages that are not finite state languages.

1.5 Finite state languages & type-3 languages: Definition of a nondeterministic finite state machine.

1.6 Examples:

1.6.1 Construction of a deterministic finite state machine from the given nondeterministic finite state machine that accepts exactly the same language.

1.6.2 Construction of a nondeterministic finite state machine that accepts the type-3 language specified by the grammar.

1.6.3 Construction of a grammar that specifies the language accepted by the finite state machine.

1.7 Analysis of algorithms: an algorithm for a problem, the time Complexity of the algorithm, & the time complexity of the problem.

1.7.1 **Algorithms & complexities:** Discussion of complexities of:

i) An algorithm for finding the largest of n numbers.

ii) An algorithm for finding the smallest of n numbers.

iii) The BUBBLE-SORT algorithm for sorting n numbers.

1.8 Tractable & Intractable problems: Definitions: efficient algorithms, inefficient algorithms, tractable & intractable problems.

Unit – 2: Numeric and generating functions **10 lectures**

2.1 Manipulation of Numeric functions: Definitions & simple examples : Numeric function, the sum, product & the convolution of two numeric functions, a scaled version & the accumulated sum of a numeric function, **t**he forward& backward difference of a numeric function, the numeric functionsS^i**a**& S^{-i}**a** for any positive integer i, where **a** is any numeric function.

2.2 Generating functions : Definition.

2.2.1 Theorem: Let A (z) & B (z) be the generating functions of the numeric functions **a** and**b** respectively. Then

i) the generating function of **a** + **b** is A (z) + B (z).

ii) the generating function of the convolution **a*b** is A (z) B (z).

iii) the generating function of the scaled version α**a** is αA (z).

iv) the generating function of S^i**a** is $z^i A(z)$.

v) the generating function of S^{-i} **a** is

$z^{-i}\ [A(z) - a_0 - a_1 z - a_2 z^2 - \ \dots - a_{i-1} z^{i-1}]$.

vi) the generating function of the forward difference of **a** is $[A(z) - a_0 - zA(z)]\ /z$.

vii) the generating function of the backward difference of **a** is $A(z) - zA(z)$.

2.2.2 Examples: Determination of numeric functions from the given generating functions.

2.3 Combinatorial problems: Definition.

2.3.1 The proof of the binomial theorem: $(1+z)^n = C(n, 0) + C(n, 1)z + C(n,2)z^2 + C(n, 3)z^3 + \ .\ .\ . + C(n, n)z^n$. by using i) the concept of generating function. ii) the combinatorial arguments.

2.3.2 The proof of the relation: $C(n, r)=C(n - 1, r) + C(n - 1, r - 1)$. By using i) the algebraic manipulation, ii) the combinatorial arguments and iii) the generating functions.

2.4 Solutions of combinatorial problems by combinatorial arguments.

Unit – 3 :Recurrence relations & recursive algorithm 11 lectures

3.1 Recurrence relations: Definitions & simple examples: recurrence relations or difference equations, solution of a recurrence relation, linear recurrence relations with constant coefficients.

3.2 Homogeneous solutions: Definition.

3.2.1 Algorithm: To find a homogeneous solution of a linear difference equation with constant coefficients & examples.

3.3 Particular solutions: Definition.

3.3.1 Method of inspection: To find the particular solution of a linear difference equation with constant coefficients & examples.

3.4 Total solutions: Definition.

3.4.1 Theorem: (statement only): The necessary & sufficient conditions for the existence of unique total solution to a k^{th} order linear difference equation.

3.4.2 Examples**:** Finding total solutions for the difference equations.

3.5 Solution by the method of generating functions: Method &examples.

3.6 Sorting algorithms: Determination of complexities of the following sorting algorithms for sorting n given numbers by using recurrence relations: i) The BUBBLESORT algorithm and ii) The Bose-Nelson algorithm.

Unit – 4: Boolean algebra **13 lectures**

4.1 Algebraic systems: Definitions & simple examples: a binary, a ternary, an m-array operation on a set A, an algebraic system.

4.2 Lattices & algebraic systems: Definitions & examples.

4.2.1 Theorem: For any a and b in a lattice (A, ≤), prove that: i) a ≤ a V b , ii) b ≤ a V b, iii) a Λ b ≤ a and iv) a Λ b ≤ b.

4.2.2 Theorem: For any a, b, c, d in a lattice (A, ≤), if a ≤ b and c ≤ d then prove that: i) a V c ≤ b V d and ii) a Λ c ≤ b Λ d.

4.3 Principle of duality: The statement of the principle of duality for lattices.

4.4 Basic properties of algebraic systems defined by the lattices:

Let (A, V, Λ) be the algebraic system defined by the lattice (A, ≤).

1) **Commutative laws:** For any two elements a & b in A,

i) a V b = b V a & ii) a Λ b = b Λ a.

2) **Associative laws:** For any three elements a, b, & c in A,

i) (a V b) V c = a V (b V c) & ii) (a Λ b) Λ c = a Λ (b Λ c).

3) **Idempotent laws:** For any element a in A, i) a V a = a ii) a Λ a = a.

4) **Absorption laws:** For any two elements a & b in A,

i) a V(a Λ b) = a & ii) a Λ(a V b) = a.

4.5 Distributive & complemented lattices: a distributive lattice, a universal lower & upper bounds of a lattice, a complement of an element. a complemented lattice.

4.5.1 Theorem: Let $(A,\leq)$ be a lattice with universal upper & lower bounds 1 & 0 respectively. Let a be any element of A. Then: $a \vee 1 = 1$, $a \wedge 1 = a$, $a \vee 0 = a$, and $a \wedge 0 = 0$.

4.5.2 Examples of lattices:

i) a lattice which is distributive but not complemented.

ii) a lattice which is complemented but not distributive.

iii) a lattice which is neither complemented nor distributive.

iv) a lattice which is both complemented and distributive.

4.5.3 Theorem: In a distributive lattice, if an element has a complement then this complement is unique.

4.6 Boolean lattices & Boolean algebras: Definitions.

4.6.1 Theorem: (De-Morgan's laws): For any a & b in a Boolean algebra,

i) $\overline{a \vee b} = \bar{a} \wedge \bar{b}$ & ii) $\overline{a \wedge b} = \bar{a} \vee \bar{b}$.

4.7 Uniqueness of finite Boolean algebras: Definitions & simple examples: a cover of an element, an atom in a Boolean algebra.

4.7.1 Theorem: In a finite Boolean algebra A,

i) For any nonzero element b there exists at least one atom a, such that $a \leq b$.

ii) If b & c are any two elements of A & if $b \wedge \bar{c} = 0$ then $b \leq c$.

iii) If b is a nonzero element and $a_1, a_2, \ldots, a_k$ be all the atoms of A such that $a_i \leq b$ then $b = a_1 \vee a_2 \vee \ldots \vee a_k$.

iv) If b is a nonzero element and $a_1, a_2, \ldots a_k$ be all the atoms of A such that $a_i \leq b$ then $b = a_1 \vee a_2 \vee \ldots \vee a_k$ is the unique way to represent b as a join of atoms.

4.7.2 Theorem: (statement only): A finite Boolean algebra A has exactly 2^n elements for some integer $n > 0$. Moreover, there is a unique Boolean algebra of 2^n elements for every $n > 0$.

4.8 Boolean functions & Boolean expressions: a Boolean expression over a Boolean algebra, the value of a Boolean expression, equivalent Boolean expressions, a Boolean function, the two-valued Boolean algebra, a minterm expression, a disjunctive normal form of an expression, a maxterm expression, a conjunctive normal form of an expression.

4.8.1 Algorithms: Let A be a two-valued Boolean algebra.

i) To obtain a Boolean expression in disjunctive normal form for a function from A^n to A.

ii) To obtain a Boolean expression in conjunctive normal form fora function from A^n to A.

4.8.2 Examples: CNF & DNF (two-valued Boolean algebras).

4.8.3 **Two-valued Boolean algebras:**

i) Boolean algebra of propositions,

ii) Boolean algebra of logic gates &

iii) Boolean algebra of switching circuits.

REFERENCE BOOKS

1. **Elements of Discrete Mathematics 2nd Edition,** C. L. Liu, Tata McGraw – Hill Publishing Company Limited New Delhi.

2. **Discrete Mathematics 2nd Edition,** Schaum's Outlines, Seymour Lipschutz, More Lars Lipson, Tata McGraw – Hill Publishing Company Limited, New Delhi.

3. **Discrete Mathematical Structures with Applications to Computer Science,** J. P. Tremblay, R. Manohar, Tata McGraw – Hill Publishing Company Limited (1974), New Delhi.

4. **Discrete Mathematics,** G. K. Ranganath and B. Sooryanarayana, S. Chand & Company LTD. 7361, Ramnagar, New Delhi 110055.

4. **Discrete Mathematical Structures 3rd edition ,** Bernard Kolman, Robert C. Busby, Sharon Ross, Prentice Hall of India private Limited, (2001), New Delhi-110 001.

Paper – XVI (B) (Special Theory of Relativity - II)

Unit – 1 : Tensors **(10 lectures)**

1.1 Introduction.

1. 1.1 Space V_n.

1.1.2 Einstein summation convention.

1.1.3 Definition : Dummy suffix , Free suffix (Real suffix)

1.1.4 Definition :Kronecker delta.

1.2 Transformation of co-ordinates.

1.2.1 Scalar (Invariant) , Vector.

1.2.2 Definition :Contravariant vector, Covariant vector.

1.2.3 Rank (order) of tensor.

1.2.4 Tensors of higher order : (a) Contravariant tensor of rank r,

(b) Covariant tensor of order r, (c) Mixed tensor of order (r + s).

1.2.5 Number of components of a tensor..

1.3 Definition: Symmetric tensor, Skew – symmetric tensor .

1.3.1 Number of distinct components of symmetric tensor and skew - symmetric tensor..

1.3.2 Results : (I) Symmetric property remains unchanged by tensor law of transformation.

(II) Anti – symmetric property remains unchanged by tensor law of transformation.

Unit – 2 : TENSOR ALGEBRA **(10 lectures)**

2.1 Addition of tensors

2.1.1 Theorem : The sum (or difference) of two tensors is a tensor of the same rank and similar character.

2.2 Contraction

2.2.1 Property :Contraction reduces the rank of a tensor by two.

2.3 Product (Multiplication)of tensors

2.3.1 Outer multiplication (Definition)

2.3.2 Inner multiplication (Definition)

2.3.3 Theorem1 : The outer product (open product) of two tensors is a tensor.

2.4 Quotient law of tensors (Definition)

2.4.1 Theorem : A set of quantities, whose inner product with an arbitrary vector is a tensor, is itself a tensor.

2.5 Definition of Reciprocal Symmetric tensor (Conjugate Tensor)

2.6 Definition of Relative Tensor, Cartesian Tensor

2.7 Definitions: Riemannian Metric, Fundamental Tensor, Associate Tensors, Raising and Lowering of suffixes

Unit – 3: TENSOR CALCULUS (10 lectures)

3.1 Definition: Christoffel Symbols of 1st kind and 2nd kind

3.1.1 Theorem: To Prove that:

i) $\overline{ij,k} + \overline{jk,i} = \frac{\partial g_{ik}}{\partial x^j}$ ii) $\Gamma^i_{ij} = \frac{\partial}{\partial x^j} \log \sqrt{(-g)}$

iii) $\Gamma^i_{ij} = \frac{\partial}{\partial x^j} \log \sqrt{g}$ iv) $\frac{\partial g^{ij}}{\partial x^k} = -g^{ij}\Gamma^j_{lk} - g^{lj}\Gamma^j_{lk}$

3.1.2 Transformation law for Christoffel symbols: Theorem: Prove that Christoffel symbols are not tensors.

3.2 Definition: Covarant derivative of a covariant vector and cotravariant vector

3.2.1 Theorem: Covarant derivative of a covariant vector is a tensor of rank 2.

3.2.2 Theorem: Covarant derivative of a cotravariant vector is a tensor.

3.2.3 Covariant differential of tensors

3.2.4 Theorem: Covariant differentiations of tensor is a tensor.

Unit – 4 : RELATIVITY AND ELECTROMAGNETISM (15 lectures)

4.1 Introduction.

4.1.1 Maxwell′s equations of electromagnetic theory in vaccum.

4.1.2 Propagation of electric and magnetic field strengths.

4.1.3 Scalar and Vector potential.

4.1.4 Four potential.

4.1.5 Transformations of the electromagnetic four potential vector.

4.2 Transformations of the charge density and current density.

4.2.1 Four current vector.

4.3 Gauge transformations.

4.4 Four dimensional formulation of the theory.

4.4.1 The electromagnetic field tensor.

4.4.2 Maxwell′s equations in tensor form.

REFERENCE BOOKS

1. Special Relativity, T. M. Karade, K. S. Adhav and Maya S. Bendre, SonuNilu , 5,Bandu Soni Layout , Gayatri Road, Parsodi, Nagpur, 440022.

2. Theory of Relativity (Special and General), J.K.Goyal , K.P.Gupta, Krishna Prakashan Media (P) Ltd., Meerut., 2006.

3. Relativity and Tensor Calculus, Karade T. M. Einstein Foundation International, 1980.

4. Mechanics, Landau L. D. and Lifshitz E. M.,Butterworth, 1998.

5. The Theory of Relativity, Moller C., Oxford University Press, 1982.

Paper XVI(C) Differential Geometry-II

(Curvature and Geodesics)

Unit – 1: LOCAL NON-INTRINSIC PROPERTIES OF A SURFACE CURVE ON A SURFACE **12 lectures**

1.1 Curvature of Normal section.

1.2 Formula for curvature of normal section in terms of fundamental magnitudes.

1.3 Definitions of normal curvature and show that these definitions are equivalent.

1.4 Meusnier′s theorem.

1.5 Examples.

1.6 Principal curvature (Definition).

1.7 The equation giving principal curvatures.

1.8 Differential equation of principal directions.

1.10 Mean curvature or Mean Normal curvature. First curvature, Gaussian curvature, Minimal surface.

Unit– 2: LINES OF CURVATURE **10 lectures**

2.1 Definition 1, Definition 2.

2.1.1 Differential equation of lines of curvature.

2.1.2 An important property of lines of curvature.

2.1.3 The differential equation of lines of curvature through a point on the surface z = f(x, y).

2.1.4 Lines of curvature as parametric curves (Theorem).

2.2 General surface of revolution.

2.2.1 Parametric curves and surface of revolution.

2.2.2 Lines of curvature on a surface of revolution.

2.2.3 Principal curvatures on surface of revolution.

2.3 Examples.

2.4 The fundamental equations of surfaces theory.

2.5 Gauss’s formulae (Theorem).

2.6 Examples.

Unit – 3: GEODESICS AND MAPPING OF SURFACES – I **13 lectures**

3.1 Geodesics.

3.2 Differential equation of Geodesics.

3.3 Necessary and Sufficient condition that the curve $v = c$ be a geodesic.

3.4 Canonical geodesic equation.

3.5 Examples.

3.6 Normal property of Geodesics.

3.7 Differential equation of Geodesic via normal property.

3.8 Examples.

3.9 Claimant′s theorem.

3.10 Examples.

3.11 Geodesic curvature.

3.12 Formulae for k_9.

3.13 Examples.

Unit – 4: GEODESICS AND MAPPING OF SURFACES – II **10 lectures**

4.1 Gauss – Bonnet theorem.

4.2 Examples.

4.3 Torsion of a Geodesic.

4.4 Examples.

4.5 Bonnet′s theorem in relation to geodesic.

4.6 Geodesic Parallels.

4.7 Geodesic polars.

4.8 Mapping of surface.

4.9 Isometric lines and isometric correspondence.

4.10 Examples.

REFERENCE BOOKS

1.Differential Geometry, Mittal and Agarwal, Krishna Prakashan Media [P] Ltd. 27th edition (1999), 11, Shivaji Road, Meerut – 1 (U.P.)

2. J. A. Thorpe, Introduction to Differential Geometry, Springer Verlag.

3. I. M. Singer and J. A. Thorpe, Lecture notes on elementary Topology and Geometry, Springer Verlag 1967.

4. B. O. Neill, Elementary Differential Geometry, Academic Press, 1966.

5. S. Sternberg, Lectures on Differential Geometry of Curves and Surfaces, **Prentice – hall 1976.**

6. D. Laugwitz, Differential and Riemannian Geometry, Academic Press, 1965.

7. R. S. Millman, and G. D. Parker, Elements of Differential Geometry Springer Verlag.

8. T. J. Willmor, An Introduction to Differential and Riemannian Geometry, Oxford University Press 1965.

Paper XVI(D) Mathematical Modeling-II

Unit–1 : MATHEMATICAL MODELLING THROUGH DIFFERENCE EQUATIONS **12 lectures**

1.1 The need for Mathematical Modeling through Difference Equations: Some Simple Models.

1.2 Basic Theory of Linear Difference Equations with constant coefficients.

1.3 Mathematical Modeling through Difference Equation in Economics.

1.4 Mathematical Modeling through Difference Equations in Population Dynamics.

Unit–2: MATHEMATICAL MODELLIG THROUGH GRAPHS **12 lectures**

2.1 Situation that can be modelled through Graphs.

2.2 Mathematical Models in terms of Directed Graphs.

2.3 Mathematical Models in terms of Signed Graphs.

2.4 Mathematical Models in terms of Weighted Diagraphs.

Unit – 3: LAPLACE TRANSFORMS AND THEIR APPLICATIONS TO DIFFERENTIAL EQUATIONS **11 lectures**

3.1 Introduction.

3.2 Properties of Laplace Transform.

3.2.1 Transforms of Derivative.

3.2.2 Transforms of Integrals.

3.3 Unit Step Functions.

3.4 Unit Impulse Functions

3.5 Application of Laplace transforms.

Unit – 4 : MATHEMATICAL MODELLING THROUGH DECAY – DIFFERENTIAL-DIFFERENCE EQUATIONS **10 lectures**

4.1 Single Species Population Models.

4.2 Prey-Predator Model.

4.3 Multispecies Model.

4.4 A Model for Growth of Population inhibited by Cumulative Effects of Pollution.

4.5 Prey- Predator Model in terms of Integro – Differential Equations.

4.6 Stability of the Prey – Predator Model.

4.7 Differential – Differences Equations Models in Relation to other Models.

REFERENCE BOOKS

1. Mathematical Modelling, J. N. Kapur, New Age International (P) Ltd., Publishers Reprint 2003.

2. Differential Equations and Their Application ,ZafarAhsan ,Prentice Hall of India , Delhi

3. Mathematical Modelling, J.G. Andrews and R. R. Mclone (1976). Butterwerths London.

4. Mathematical Modelling Techniques, R. Aris (1978) , Pitman.

5. Differential Equation Models, Martin Braun, C. S. Coleman, D.A.Drew , Vol. 1.

6. Political and Related Models, Steven J. Drams, Kl. F Lucas, P. D. Straffin (Eds),Vol. 1.

7. Discrete and System Models, W. F. Lucas, F. S. Roberts, R. M. Thrall, Vol. 3.

8. Life Science Models, H. M. Roberts And M. Thompson, Vol. 4.

9. “ Thinking with Models ” (Mathematical Models in Physical, Biological and Social Sciences), T. Saaty and J.AlexanderPergamon Press, New York.

Paper XVI(E) (Applications of Mathematics in Insurance)

Unit – 1 :INSURANCE FUNDAMENTALS **20 lectures**

1.1 Insurance defined.

1.2 Meaning of loss. Chances of loss, peril, hazard and proximate cause in insurance.

1.3 Costs and benefits of insurance to the society and branches of insurance-life insurance and various types of general insurance.

1.4 Insurable loss exposures features of a loss that is ideal for insurance.

Unit – 2 :LIFE INSURANCE MATHEMATICS. **5 lectures**

2.1 Construction of Mortality Tables.

2.2 Computation of Premium of Life Insurance for a fixed duration and for the whole life.

Unit – 3 :DETERMINATION OF CLAIMS FOR GENERAL INSURANCE
10 lectures

3.1 Determination of claims for general insurance using Poisson distribution.

3.2 Determination of claims for general insurance using and Negative Binomial Distribution.

3.2.1 The Polya Case.

Unit – 4 :DETERMINATION OF THE AMOUNT OF CLAIMS IN GENERAL INSURANCE **10 lectures**

4.1 Compound Aggregate claim model and its properties and claims of reinsurance.

4.2 Calculation of a compound claim density function.

4.3 F-recursive and approximate formulae for F.

REFERENCE BOOKS

1. Corporate Finance -Theory and Practice, AswathDamodaran, John Wiley & Sons.Inc.

2. Options, Futures, and Other Derivatives, John C. Hull, Prentice - Hall of India Private Limited.

3. An Introduction to Mathematical Finanace, Sheldon M . Ross, Cambridge University Press.

4. Introduction to Risk Management and Insurance, Mark S. Dorfman, Prentice Hall, Englwood Cliffs, New Jersey

PAPER NO XVI (F) MECHANICS-II

1. **Rectilinear motion:** Velocity, acceleration, radial and transverse components of velocity and acceleration, Newton's first law, mass, force, Newton's second law, weight, impulse, work, kinetic energy, potential energy, conservation of energy, rectilinear motion of a particle with uniform acceleration. **12 lectures**
2. **Projectile motion:** Definitions-point of projection, velocity of projection, angle of projection, horizontal range, time of flight, range on an inclined plane, equation of the path of a projectile, examples. **12 lectures**
3. Constrained motion: components of velocity and acceleration parallel to the coordinate axes, tangential and normal components of acceleration, motion of a heavy particle on a smooth curve in a vertical plane, motion on a smooth vertical circle. **10 lectures**
4. Central orbits: motion under inverse square law, to find the law of force, determination of orbits given the law of force, Kepler's laws of planetary motion. **11 lectures**

REFERENCE BOOKS

1. Dynamics by A.S.Ramsey, CBS Publishing and distributors
2. Mechanics by B.Singh, S.K. Pundir, P.K.Sharma, Pragatiprakashan
3. Dynamics of particle by Vasistha and Agarwal ,Krishna prakashan A text book of dynamics by M. Ray

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REVISED SYLLABUS OF B. Sc. Part III

MATHEMATICS (Practical)

Implemented from June – 2012

Computational Mathematics Laboratory - IV

(Operations Research Techniques)

Sr. No.	Topic	No. Of Practicals
	Linear Programming :	
1	Simplex Method : Maximization Case	1
2	Simplex Method : Minimization Case	1
3	Two-Phase Method	1
4	Big-M-Method	1
	Transportation Problems :	
5	North- West Corner Method	1
6	Least Cost Method	1
7	Vogel's Approximation Method	1
8	Optimization of T.P. by Modi Method	1
	Assignment Problems:	
9	Hungarian Method	1
10	Maximization Case in Assignment Problem	1
11	Unbalanced Assignment Problems	1
12	Traveling Salesman Problem	1
	Theory of Games:	
13	Games with saddle point	1
14	Games without saddle point : (Algebraic method)	1
15	Games without saddle point : a) Arithmetic Method b) Matrix Method	1
16	Games without saddle point : Graphical method	1

REFERENCE BOOKS

1. Operations Research [Theory and Applications], By J.K.Sharma
   Second edition, 2003, Macmillan India Ltd., New Delhi.
2. Operations Research: S. D. Sharma.

Computational Mathematics Laboratory V

[Laplace Transform]

Sr. No.	Topic	Number of Practicals
-	**I - Laplace Transform**	
1	Laplace Transform of Elementary Functions	1
-	**II - Properties of Laplace Transform**	
2	Laplace Transform by using Linearity Property	1
3	Laplace Transform by using (i) First Translation Property (ii) Second Translation Property (iii) Change of Scale Property	 1
4	i) Laplace Transform of Multiplication by t^n ii) Laplace Transform of Division by t	1
5	i) Laplace Transform of Derivatives ii) Laplace Transform of Integrals	1
6	Laplace Transform of Periodic Functions	1
-	**III - Inverse Laplace Transform**	
7	Inverse Laplace Transform of Elementary Functions	1
-	**IV - Properties of Inverse Laplace Transform**	
8	Inverse Laplace Transform by using Linearity Property	1
9	Inverse Laplace Transform by using (i) First Translation Property (ii) Second Translation Property (iii) Change of Scale Property	1
10	(i) Inverse Laplace Transform of Multiplication by S (ii) Inverse Laplace Transform of Divistion by S	1
11	(i) Inverse Laplace Transform of Derivatives (ii) Inverse Laplace Transform of Integrals	1
12	The Convolution Theorem	1
13	HEAVISIDE Expansion Formula : Set (I)	1
14	HEAVISIDE Expansion Formula : Set (II)	1
-	**V - Applications**	
15	Applications of Ordinary Differential Equation with Constants Coefficients : Set (I)	1
16	Applications to Ordinary Differential Equations with Variable Coefficients	1

REFERENCE BOOKS

1. Differential Equations : Kulkarni, Jadhav, Patwardhan, Kubade, Phadake Prakashan, Kolhapur.
2. Integral Transform : Dr. J.K. Goyal, K.P. Gupta. Pragati Prakashan, Meerut.
3. Integral Transform : Vasishtha, Gupta Krishna Prakashan Meerut.

Computational Mathematics Laboratory – VI

(Numerical Recipes in C++, Matlab & Microsoft Excel)

Sr.No.	Topic	No. of Practical
1	***C++ Introduction:*** History, Identifiers, Keywords, constants, variables, C++ operations. Data types in C++: Integer, float, character. Input/Output statements, Header files in C++, iostream.h, math.h etc.	1
2	***Expressions in C++:*** (i) constant expression, (ii) integer expression, (iii) float expression, (iv) relational expression,(v) logical expression. Declarations in C++.Program Structure of C++ .Simple illustrative programs. ***Control Statements:*** (a) if, if – else, nested if. (b) for loop, while loop, do-while loop. (c) break, continue, goto, switch statements. Simple Programs – 1) Euclid's algorithm to find gcd and then to find lcm of two numbers a, b. 2) To list 1!, 2!, 3!, … , n!. 3) To print prime numbers from 2 to n. etc.	1
3	***Arrays :*** (a) Sorting of an array. (b) Linear search. (c) Binary search. (d) Matrix multiplication	1
4	***Functions:*** User defined functions of four types with illustrative programs such as factorial of non-negative integers etc.	1
5	***Microsoft Excel Knowledge*** *The student is expected to familiarize with Microsoft-Excel software for numerical Computations*. Opening Microsoft Excel, Overview of Excel Naming parts of the Excel Window, File New, File Open, File Close, File Save/Save As, Auto Fill and Data Series, Cut, Copy, Paste, Insert, Menu Bar, Toolbar, Right clicking, Fill Handle, Inserting, deleting, and moving, Rows, Columns, Sheets, Mathematical symbols (Preset Functions) AutoSum, Copying a calculation using the fill handle, Formula Bar, Editing Formula Using preset functions, Order of operations, Print a worksheet. 1) Mean and S.D. of raw data, arrange given numbers in ascending or descending order. 2) Find the inverse of Matrix, transpose of matrices, determinant of square matrix, addition, multiplication of matrices.	3
6	***MATLAB Knowledge*** *The student is expected to familiarize with MATLAB software for numerical computation.* Basic of MATLAB, Tutorial Lessons.	3

	1) Using MATLAB functions find the inverse of Matrix, transpose of matrices, determinant of square matrix, addition, transpose and multiplication of matrices, eigen values, and eigen vectors. 2) Using MATLAB creating graphs of simple functions	
Numerical Methods **-** Find the solutions of following methods either using C++ programming **OR** M-Excel **OR** MATLAB software		
7	Interpolation : (a) Lagrange′s interpolation formula. (b) Newton Gregory forward interpolation formula. (c) Newton Gregory backward interpolation formula.	2
8	Numerical Methods for solution of A system of Linear Equations: (Unique solution case only) (a) Gauss – Elimination Method. (b) Gauss – Jordan Method.	2
9	Numerical Methods for solution of Ordinary Differential Equations: (a) Euler Method (b) Euler′s Modified Method (c) Runge Kutta Second and Fourth order Method.	2

REFERENCE BOOKS

1. Programming with C++, D. Ravichandran Second Edition, Tata Mac- Graw- Hill publishing Co. Ltd., New Delhi (2006).
2. Working with Excel 97 A Hands on Tutorial, Tata Mac- Graw- Hill Series, publishing Co. Ltd., New Delhi (2006).
3. Getting Started with MATLAB 7, Rudra Pratap, OXFORDUNIVERSITY PRESS.(2009)
4. Numerical Technique Lab MATLAB Based Experiments, K. K. Mishra, I. K. International Publishing House Pvt. Ltd., New Delhi.(2007)
5. MATLAB An Introduction with Applications, Amos Gilat, S. P. Printers, Delhi.(2004)

Computational Mathematics Laboratory – VII

(Project Work, Study Tour Report, Viva- Voce)

A. <u>PROJECT</u>: **[30 Marks]**

Each student of B.Sc. III (Mathematics) is expected to read, collect, understand culture of Mathematics, its historic development. He is expected to get acquainted with Mathematical concepts, innovations, relevance of Mathematics. Report of the project work should be submitted through the respective Department of Mathematics.

Topics for Project work :

1. Contribution of the great Mathematicians such as Rene Descart, Leibnitz, Issac Newton, Euler, Lagrange, Gauss, Riemann, Fourier, Bhaskaracharya, Srinivas Ramanujan etc.
2. On the following topics or on other equivalent topics :
   - (i) Theorem on Pythagoras and Pythagorean triplets.
   - (ii) On the determination of value of π.
   - (iii) Remarkable curves.
   - (iv) Orthogonal Latin Spheres.
   - (v) Different kinds of numbers.
   - (vi) Law of quadratic reciprocity of congruence due to Gauss.
   - (vii) Invention of Zero.
   - (viii) Vedic Mathematics.
   - (ix) Location of objects in the celestial sphere.
   - (x) Kaprekar or like numbers.
   - (xi) Playing with PASCAL′S TRIANGLE and FIBONACCI NUMBERS.
   - (xii) PERT and CPM.
   - (xiii) Magic squares.
   - (xiv) Software such as Mupad, Matlab, Mathematica, Xplore, etc.
   - (xv) Pigeon hole principle.

Evaluation of the project report will be done by the external examiners at the time of annual examination.

B. STUDY TOUR **[5 Marks]**

List of suggested places : Banglore, Goa (Science Center), Pune, Kolhapur, Mumbai, Ahamdabad, Hydrabad, etc.

C. VIVA-VOCE (on the project report). **[15 Marks]**

REFERENCE BOOKS

1. The World of Mathematics, James R. Newman & Schuster, New York.

2. Men Of Mathematics, E.T.Bell.

3. Ancient Indian Mathematics, C. N. Srinivasayengar.

4. Vedic Mathematic , Ramanand Bharati.

5. Fascinating World of Mathematical Science Vol. I, II, J. N. Kapur.

JOURNALS

1. Mathematical Education.

2. Mathematics Today.

3. Bona Mathematical.

4. Ramanujan Mathematics News Letter.

5. Resonance.

6. Mathematical Science Trust Society (MSTS), New friends′ colony, New Delhi 4000 065.

<u>SCHEME OF TEACHING</u> :

a) Theory

Paper-No.	**Title of the Paper**	**Total Marks**	**Periods (Theory per Paper) per week**
	(Semester V) Compulsory Papers		3 Lectures
IX	**Real Analysis**	50 (40 + 10)	
X	**Modern Algebra**	50 (40 + 10)	
XI	**Partial Differential Equations**	50 (40 + 10)	
	(Semester V) Optional Papers		
XII(A)	**Symbolic Logic & Graph Theory**	50 (40 + 10)	
XII(B)	**Special Theory of Relativity - I**	50 (40 + 10)	
XII(C)	**Differential Geometry - I**	50 (40 + 10)	
XII(D)	**Mathematical Modelling - I**	50 (40 + 10)	
XII(E)	**Applications of Mathematics to Finance**	50 (40 + 10)	
XII(F)	**Mechanics - I**	50 (40 + 10)	
	(Semester VI) Compulsory Papers		
XIII	**Metric Spaces**	50 (40 + 10)	3 Lectures
XIV	**Linear Algebra**	50 (40 + 10)	
XV	**Complex Analysis**	50 (40 + 10)	
	(Semester VI) Optional Papers		
XVI(A)	**Boolean Algebra & Algorithms**	50 (40 + 10)	
XVI (B)	**Special Theory of Relativity - II**	50 (40 + 10)	
XVI (C)	**Differential Geometry - II**	50 (40 + 10)	
XVI (D)	**Mathematical Modelling - II**	50 (40 + 10)	
XVI (E)	**Applications of Mathematics to Insurance**	50 (40 + 10)	
XVI (F)	**Mechanics - II**	50 (40 + 10)	

b) Practical (Annual)

Paper-No.	Title of the Paper	Total Marks	Periods (Practical) per week
	(Semester V & VI) Annual Pattern		
CML - IV	Operations Research Techniques	200	5*
CML - V	Laplace Transform		5*
CML - VI	Numerical Recipes in C++, Matlab & Microsoft Excel		5*
CML - VII	Project Work, Study Tour, Viva - Voce		5*

* Note : 20 period per week per batch (**Batch as a whole class**).

Work - Load

(i) Total teaching periods for **Paper – IX , X, XI, XII** (Semester – V) are **12 (Twelve)** per week. (3 periods per paper per week) and total teaching periods for **Paper – XIII, XIV, XV,XVI** (Semester – VI) are **12 (Twelve)** per week. (3 periods per paper per week)

(ii) Total teaching periods for Practical Course in Mathematics – **CML- IV, V, VI & VII, 20 (Twenty)** per week per batch (**Batch as a whole class**)

Scheme of examination

The Theory examination shall be conducted at the end of each semester.

The Theory paper shall carry 40 Marks.

There will be 10 internal marks per paper per semester

The practical examination shall be conducted at the end of each year.

Per CML shall carry 50 marks.

The evaluation of the performance of the students in theory shall be on the basis of examination.

NATURE OF QUESTION PAPER THEORY COMMON MENTIONED SPERATELY:

Nature of Practical Question Papers

I] For Computational Mathematics Lab IV, V and VI : Marks 50

Distribution of Marks

1. University Exam: **Marks 40** 2. Journal: **Marks 10**

Q. No 1 (A) ……………………………………..... **10 Marks**

OR

Q. No 1 (A) ……………………………………..... **10 Marks**

Q. No 1 (B) ………………………………………. **05 Marks**

OR

Q. No 1 (B) ………………………………………. **05 Marks**

Q. No 2 (A) ……………………………………..... **10 Marks**

OR

Q. No 2 (A) ……………………………………..... **10 Marks**

Q. No 2 (B) ………………………………………. **05 Marks**

OR

Q. No 2 (B) ………………………………………. **05 Marks**

Q. No 3 Short Answers [Any **two** out of **four**] **10 Marks**

II For computational lab VII : Marks 50

1) Project : 30 Marks
2) Viva-voce : 15 Marks
3) Tour Report : 05 Marks

Standard of passing

As prescribed under rules and regulation for each degree program.

Requirements

Qualifications for Teacher

M.Sc. Mathematics (with NET /SET as per existing rules)

Equipments-

1) Calculators	:	20
2) Computers	:	10
3) Printers	:	01

License software's- O/S , Application S/W , Packages S/W as per syllabus.
